

✓ MODELING AND OPTIMAL PARAMETER ESTIMATION OF SOLID STATE
POLYMERIZATION OF NYLON 6

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MASTER OF TECHNOLOGY

by
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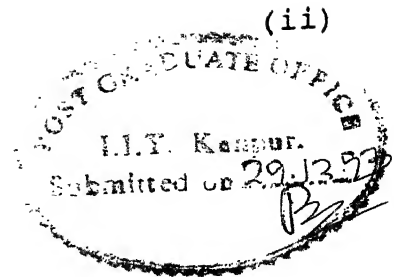
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CERTIFICATE



This is to certify that the present work 'MODELING AND OPTIMAL PARAMETER ESTIMATION OF SOLID STATE POLYMERIZATION OF NYLON 6' has been carried out by Mr. Mukund R. Kulkarni under my supervision and that this has not been submitted elsewhere for a degree.

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Mukund R. Kulkarni

**Dedicated
to
My Parents**

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NOMENCLATURE

| | |
|-----------------------------------|--|
| -A | Polymer molecule with an unreacted A group. |
| -B | Polymer molecule with an unreacted B group. |
| C_1 | ϵ -Caprolactam. |
| D_m, D_p, D_s | Overall (micro-level) diffusivity of monomer, polymer, solvent molecule respectively, m^2/hr . |
| $D_{m,ref}, D_{p,ref}, D_{s,ref}$ | Overall diffusivity of monomer, polymer and solvent molecule at reference condition, m^2/hr . |
| D_w | Diffusivity (macro-level) of water inside the reacting pellet, m^2/hr . |
| k_i | Overall forward rate constant of i th reaction in the solid phase, $kg/mol-hr$. |
| k'_i | Overall reverse rate constant of i th reaction in the solid phase, $kg/mol-hr$ or hr^{-1} . |
| $k_{i,o}$ | Intrinsic forward rate constant of i th reaction in the solid phase, $kg/mol-hr$. |
| $k'_{i,o}$ | Intrinsic reverse rate constant for i th reaction in the solid phase ($= k_{i,o}/K_i$), $kg/mol-hr$ or hr^{-1} . |
| K_i | Equilibrium constant of i th reaction. |
| K_s | Parameter in equation for glass transition temperature of polymer, K^{-1} . |
| $(MW_m), (MW_s)$ | Molecular weights of pure monomer and solvent, $kg\ mol^{-1}$. |
| M_{jp} | Molecular weight of jumping polymer unit, $kg\ mol^{-1}$. |
| P_n | Polymer molecule of chain length n . |

| | |
|---|--|
| [P] | Concentration of polymer molecules P of all lengths, mol/kg. |
| \bar{Q} | Spatially-averaged polydispersity index $\equiv (\bar{\lambda}_2/\bar{\lambda}_1)/(\bar{\lambda}_1/\bar{\lambda}_0).$ |
| r_D | Radius at which the concentration of polymer molecule P approaches the bulk value, m. |
| $r_{m,j}$ | Radius swept by a polymer molecule having one unreacted A group fixed at one end, m. |
| R | Radius of the solid pellet, m. |
| T | Reaction temperature, K. |
| T_g | Glass transition temperature, K. |
| $T_{g\infty}$ | Glass transition temperature at infinite chain length, K. |
| $\hat{V}_m^*, \hat{V}_p^*, \hat{V}_s^*$ | Specific critical hole free volume of monomer, polymer, or solvent, m ³ /kg |
| $\hat{V}_m^o, \hat{V}_p^o, \hat{V}_s^o$ | Specific volumes of monomer, polymer, or solvent, m ³ /kg |
| V_{fm}, V_{fs}, V_{fp} | Free volume fraction of monomer, solvent and polymer respectively inside the reacting pellet. |
| $[W]_s$ | Surface concentration of water, mol/kg. |

GREEK LETTERS

| | |
|------------|--|
| α_g | Coefficient of thermal expansion of polymer in glassy state, K ⁻¹ . |
| α_l | Coefficient of thermal expansion of polymer in liquid state, K ⁻¹ . |
| γ | Overlap factor |

$\theta_1, \theta_2, \theta_3$ Adjustable parameters in the model, hr.

λ_k kth moment of all polymer molecules, $\lambda_k = \sum_{n=1}^{\infty} n^k [P_n]$;
 $k = 0, 1, 2, \dots$, mol/kg.

$\bar{\lambda}_k$ Spatial average value of $\lambda_k = \frac{\int_0^R r^2 \lambda_k dr}{\int_0^R r^2 dr}$

μ_n Number average chain length $\equiv \lambda_1 / \lambda_0$.

$\bar{\mu}_n$ Spatial average value of $\mu_n = \frac{\int_0^R r^2 \lambda_1 dr}{\int_0^R r^2 \lambda_0 dr} \equiv \frac{\bar{\lambda}_1}{\bar{\lambda}_0}$

ζ_j Parameter used in the forward rate constant for the jth reaction in solid phase, $\text{kg}^2/\text{mol}^2\text{-hr}$.

ξ_{13}, ξ_{23} Ratio of volume of a monomer (or solvent) molecule to the volume of a polymer jumping unit

ρ_m, ρ_p, ρ_s Density of pure (liquid) monomer, polymer or solvent at temperature T (at time t), kg m^{-3}

ρ_m^0, ρ_s^0 Density of pure (liquid) monomer or solvent at initial temperature, kg m^{-3}

ϕ_m, ϕ_p, ϕ_s Volume fractions of monomer, polymer or solvent in liquid at time t

χ as defined in Table II

SUBSCRIPTS

b bulk value
 m monomer (ϵ caprolactam)
 p polymer
 s solvent (water)

ABSTRACT

An improved mathematical model using the Vrentas-Duda theory for diffusion coefficients is developed for solid state polymerization (SSP). This model is applied to nylon 6, and best-fit values of the parameters are obtained for this system using the Box-complex analysis on available experimental data. The sensitivity of results to variations of these parameters is studied. It is found that the ring opening reaction can easily be omitted, and the number of parameters thereby decreased. Detailed quantitative results are obtained to study the effects of changing the important operating conditions on SSP, e.g., intermediate remelting of nylon 6 powder, value of the water concentration (or level of vacuum) in the vapor phase, leaching of monomer and water before SSP, size and degree of crystallinity of polymer particles, etc. The diffusional effects on the rate constants are found to depend upon a complex interplay of several important factors like the generation of monomer and water by reaction, removal of water by macro-level diffusion, changes in the intrinsic rate constants due to decrease in the concentration of polymer molecules, etc.

CHAPTER 1

INTRODUCTION

It is often difficult to obtain extremely high molecular weight polymers by conventional melt polycondensations because of high melt viscosities and limitations posed by equilibrium. However, it is possible to raise the degree of polymerization, DP, of several step growth polymers by heating them in vacuum or in an inert gas at temperatures below their melting points but above their glass transition temperatures. This method, known as solid state polymerization (SSP) is quite commonly used in industry. Applications involving stringent operating conditions (e.g., tire cords) and certain processing techniques like blow molding and extrusion require such polymers. Both batch as well as continuous operations are being practiced in industry for this purpose, involving the use of fixed bed, rotary drum and fluidized bed reactors.

Studies on SSP go back to the early forties. Most of the early information is in the form of patents. Work on SSP started getting reported in the open literature about two decades ago.¹⁻⁴ Gaymans et al.⁵ carried out a systematic experimental study on the SSP of nylon 6, while Griskey and Lee⁴ and Chen et al.¹ provided some experimental data on nylon 66 and on nylon 6-10 and polyethylene terephthalate respectively, under a variety of conditions. Pilati⁶ and Fakirov⁷ have provided excellent reviews of the work in this area.

Most of the early studies provided semi-empirical models to explain experimental data. Kumar et al.⁸ proposed a phenomenological model having a molecular basis to explain some

important trends observed in the SSP of condensation polymers. They adapted the model of Chiu et al.⁹ for the gel effect in free radical polymerizations, to SSP. More recently, Kaushik and Gupta¹⁰ extended and improved this model and applied it to the SSP of nylon 6. They obtained curve-fitted values of the parameters in their model, using the experimental data of Gaymans et al.⁵ In the present study, we present a new model, which uses the free volume theory of Vrentas and Duda¹¹ to account for the changes in the diffusion coefficients more accurately, using parameters which can be obtained directly from independent experiments which do not involve reactions. This model, applicable to any reversible step growth polymerization in the presence of diffusional limitations, is applied to the specific case of the solid state polymerization of nylon 6 in this work. An optimal parameter estimation package is again used to obtain the best-fit values of the parameters used in this model, using the experimental data of Gaymans et al.⁵

CHAPTER 2

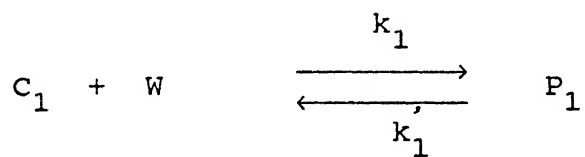
FORMULATION

There are three major reversible reactions in the kinetic scheme of nylon 6 polymerization¹²: ring opening, polycondensation and polyaddition. These are shown in Table I. The reaction most susceptible to diffusional limitations is the forward step of the polycondensation reaction, since it involves the diffusion of two large molecules towards each other. In SSP however, it is expected that the diffusion of smaller molecules like water and ϵ -caprolactam can also be affected. The present model takes this also into consideration. Thus, the reactions which are considered to be diffusion controlled in the present study are the forward and reverse steps of polycondensation, as well as the forward step of the polyaddition reaction, since at least one large molecule is involved in these three reactions. The backward steps of ring opening and polyaddition reactions are not diffusion controlled, since they involve only a single species. Similarly, the forward step of the ring opening reaction is assumed not to be diffusion controlled since it involves the diffusion of two small molecules.

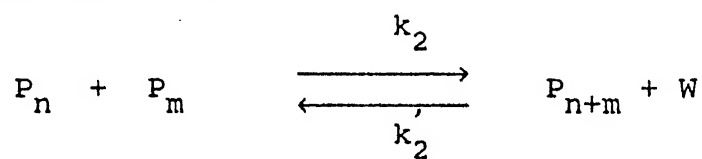
We now model the variation of the forward rate constant of the polycondensation reaction associated with diffusional limitations. The approach used is similar to that suggested by Achilias and Kiparissides.^{13,14} Fig. 1 shows a polymer chain, P_m , with its functional A group at the center of a hypothetical sphere of radius $r_{m,1}$. Within this sphere, the effective concentration of polymer molecules is $[P]_m$. At a large distance, r_D , from the central functional group, the concentration of polymer molecules

TABLE I
KINETIC SCHEME FOR POLYMERIZATION OF NYLON 6

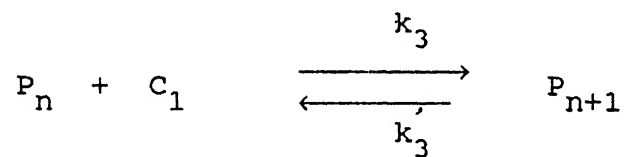
Ring Opening



Polycondensation



Polyaddition



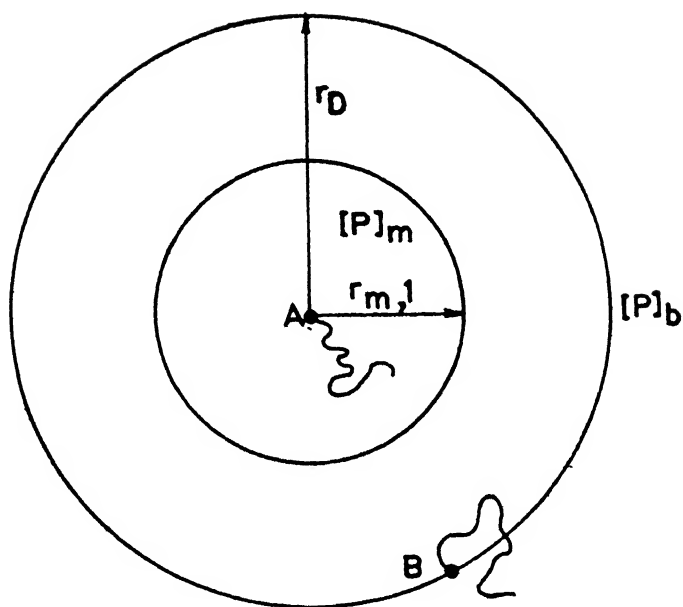


FIG. 1

Segmental diffusion of a polymer molecule, P_n , with an unreacted B group toward another polymer molecule, P_m , with an unreacted A group.

approaches the (local) bulk value, $[P]_b$. The reaction between the central functional A group and a B group of another polymer molecule, P_n , far away, occurs only after the latter comes in the vicinity of A by diffusion. The diffusion rate can be easily shown⁹ to be independent of the radial location, and is given by

$$4 \pi r^2 D_p \frac{d[P]}{dr} = C_1 \quad (1)$$

where D_p is the micro-level diffusivity of the polymer molecule in the medium. The following boundary conditions apply

$$\text{at } r = r_{m,1} \quad [P] = [P]_m \quad (a)$$

$$\text{at } r = r_D \rightarrow \infty \quad [P] = [P]_b \quad (b) \quad (2)$$

Eqns. (1) and (2) give

$$4 \pi D_p r_{m,1} ([P]_b - [P]_m) = C_1 \quad (3)$$

The rate of consumption of functional group B within the sphere of radius $r_{m,1}$ because of reaction between two large molecules (forward reaction of polycondensation step) is

$$\frac{4}{3} \pi r_{m,1}^3 (k_{2,o} [P]_m [P]_b) = C_1 \quad (4)$$

where $k_{2,o}$ is the intrinsic rate constant for this reaction. Eqns. (3) and (4) give the expression for the effective polymer concentration at $r = r_{m,1}$ as

$$[P]_m = \frac{[P]_b}{1 + k_{2,o} \left\{ \frac{r_{m,1}^2}{3D_p} \right\} [P]_b} \quad (5)$$

The overall rate of this reaction can also be written as $k_2[P]_b^2$, where k_2 is the apparent rate constant for this reaction, accounting for diffusional limitations as well. The two expressions for the rate give

$$k_2 [P]_b^2 = k_{2,o} [P]_m [P]_b \quad (6)$$

On substituting the expression for $[P]_m$ from Eqn. (5) in Eqn. (6), we obtain

$$\frac{1}{k_2} = \frac{1}{k_{2,o}} + \left(\frac{r_{m,1}^2}{3 D_p} \right) [P]_b \quad (7)$$

where $[P]_b$, the local bulk polymer concentration, can be rewritten in terms of the molecular weight distribution and its moments (see nomenclature) as

$$[P]_b = \sum_{n=1}^{\infty} [P_n]_b = \lambda_{o,b} \quad (8)$$

An analogous mathematical treatment for the backward step of the polycondensation reaction and the forward step of the polyaddition reaction leads to

$$\frac{1}{k'_2} = \frac{1}{k'_{2,o}} + \left(\frac{r_{m,2}^2}{3 D_w} \right) [P]_b \quad (a)$$

$$\frac{1}{k_3} = \frac{1}{k_{3,o}} + \left(\frac{r_{m,3}^2}{3 D_m} \right) [P]_b \quad (b) \quad (9)$$

where D_w and D_m are the effective micro-level diffusivities of water and monomer in the medium.

The free volume theory of Vrentas and Duda¹¹ can now be used for the diffusivities. The effective diffusivity, D_p , of polymer

molecules can be written as

$$D_p = D_{po} \mu_n^{-2} \exp\left(-\frac{E_p}{R/T}\right) \exp(-\chi) \quad (10)$$

where

$$\chi = \frac{\gamma \left\{ \frac{\rho_m \phi_m \hat{V}_m^*}{\xi_{13}} + \frac{\rho_s \phi_s \hat{V}_s^*}{\xi_{23}} + \rho_p \phi_p \hat{V}_p^* \right\}}{\rho_m \phi_m \hat{V}_m^* v_{fm} + \rho_s \phi_s \hat{V}_s^* v_{fs} + \rho_p \phi_p \hat{V}_p^* v_{fp}} \quad (11)$$

The same notation has been used here as in Ref. 13 (also see nomenclature). A semiempirical procedure is now used to simplify Eqn. (10) further. A reference state is defined as $\phi_m \rightarrow 0$, $\phi_s \rightarrow 0$, and $\phi_p \rightarrow 1$. Eqn. (10) can be written in terms of the reference state (ref) as

$$\begin{aligned} \frac{1}{k_2} &= \frac{1}{k_{2,o}} + \left(\frac{r_{m,1}^2}{3 D_{p,ref}} \right) \lambda_o \frac{1}{(D_p / D_{p,ref})} \\ &= \frac{1}{k_{2,o}} + \left(\frac{r_{m,1}^2}{3 D_{p,ref} \mu_{n,ref}^2} \right) \mu_n^2 \lambda_o \exp[\chi - \chi_{ref}] \\ &\equiv \frac{1}{k_{2,o}} + \theta_1(T) \mu_n^2 \lambda_o \exp[\chi - \chi_{ref}] \end{aligned} \quad (12)$$

where

$$\chi_{ref} = \frac{\gamma}{V_{fp}} \quad (13)$$

The parameter, $\theta_1(T)$, is a function of temperature alone¹³ for a given sample (it depends on the degree of crystallinity through the parameter $D_{p,ref}$). In a similar manner, Eq. (9) can be written as

$$\frac{1}{k_2'} = \frac{1}{k_{2,o}'} + \theta_2(T) \lambda_o \exp [\xi_{23}(\chi - \chi_{\text{ref}})] \quad (a)$$

$$\frac{1}{k_3} = \frac{1}{k_{3,o}} + \theta_3(T) \lambda_o \exp [\xi_{13}(\chi - \chi_{\text{ref}})] \quad (b) \quad (14)$$

Table II summarizes the final equations for the apparent rate constants. Numerical values of and correlations for the various parameters required have been compiled from various sources and are given in Table III along with references.^{8,13-22} These parameters have been determined experimentally using independent measurements on nonreacting nylon 6 systems. The glass transition temperature of nylon 6 has been assumed to be a function of its spatially averaged value of the number average chain length, $\bar{\mu}_n$, as assumed earlier^{8,10}.

The micro-level diffusivity of water leads to a difference in the water concentrations at $r = r_{m,2}$ (where the concentration is $[W]_m$) and $r = r_D \rightarrow \infty$ (where the concentration is $[W]_b$). This gradient is accounted for in the expression for the apparent rate constant, k_2' , in Eqn. (14a). It may be pointed out that the bulk concentration of water, $[W]_b$, itself is not constant, but varies with location due to the macro-level diffusion of water through the polymer pellet to the flowing inert gas outside. The diffusional resistance of W in the gas phase is neglected in this formulation. However, it is expected that there is considerable resistance for the diffusion of water within the pellet. We consider the reacting pellets to be spherical with the condensation byproduct, W, diffusing radially outwards. The macro-level diffusivity, D_w , of W through the 'solid' nylon 6

TABLE II
APPARENT REACTION RATE CONSTANTS

$$\frac{1}{k_2} = \frac{1}{k_{2,o}} + \theta_1(T) \mu_n^2 \lambda_o \frac{1}{\exp[-\chi + \chi_{\text{ref}}]}$$

$$\frac{1}{k'_2} = \frac{1}{k'_{2,o}} + \theta_2(T) \lambda_o \frac{1}{\exp[\xi_{23}(-\chi + \chi_{\text{ref}})]}$$

$$\frac{1}{k_3} = \frac{1}{k_{3,o}} + \theta_3(T) \lambda_o \frac{1}{\exp[\xi_{13}(-\chi + \chi_{\text{ref}})]}$$

$$k_1 = k_{1,o}; \quad k'_1 = k'_{1,o} = k_{1,o}/K_1$$

$$k'_3 = k'_{3,o} = k_{3,o}/K_3; \quad k'_{2,o} = k_{2,o}/K_2$$

$$\chi = \frac{\gamma \left\{ \frac{\rho_m \phi_m \hat{V}_m^*}{\xi_{13}} + \frac{\rho_s \phi_s \hat{V}_s^*}{\xi_{23}} + \rho_p \phi_p \hat{V}_p^* \right\}}{\rho_m \phi_m \hat{V}_m^* v_{fm} + \rho_s \phi_s \hat{V}_s^* v_{fs} + \rho_p \phi_p \hat{V}_p^* v_{fp}}$$

$$\chi_{\text{ref}} = \frac{\gamma}{v_{fp}}$$

$$\xi_{13} = \frac{\hat{V}_m^* (MW_m)}{\hat{V}_p^* M_{jp}}$$

$$\xi_{23} = \frac{\hat{V}_s^* (MW_s)}{\hat{V}_p^* M_{jp}}$$

$$\rho_m = 1 / \hat{V}_m^o(T); \quad \rho_p = 1 / \hat{V}_p^o(T); \quad \rho_s = 1 / \hat{V}_s^o(T)$$

Contd...

(Table II contd...)

$$\phi_m = \frac{[C_1] (MW_m) \hat{V}_m^o}{[C_1] (MW_m) \hat{V}_m^o + \lambda_1 (MW_m) \hat{V}_p^o + [W] (MW_s) \hat{V}_s^o}$$

$$\phi_p = \frac{\lambda_1 (MW_m) \hat{V}_p^o}{[C_1] (MW_m) \hat{V}_m^o + \lambda_1 (MW_m) \hat{V}_p^o + [W] (MW_s) \hat{V}_s^o}$$

$$\phi_s = \frac{[W] (MW_s) \hat{V}_s^o}{[C_1] (MW_m) \hat{V}_m^o + \lambda_1 (MW_m) \hat{V}_p^o + [W] (MW_s) \hat{V}_s^o}$$

p : polymer

s : solvent (water)

m : monomer (ϵ -caprolactam)

local bulk values of concentrations and moments used for all cases (subscript b not indicated)

TABLE III
PARAMETERS USED TO CALCULATE THE APPARENT REACTION RATE CONSTANTS

| | | | |
|----------------------------------|---|--|----------------------------|
| (MW_m) | = | 0.11316 kg/mol | |
| (MW_s) | = | 0.018 kg/mol | |
| \hat{V}_m^* | = | $8.64 \times 10^{-4} \text{ m}^3/\text{kg}$ | (Ref. 16) ⁺ |
| $\hat{V}_m^o(190^\circ\text{C})$ | = | $11.325 \times 10^{-4} \text{ m}^3/\text{kg}$ | (Ref. 16) ⁺ |
| \hat{V}_p^* | = | $8.123 \times 10^{-4} \text{ m}^3/\text{kg}$ | (Ref. 17) ⁺ |
| $\hat{V}_p^o(T)$ | = | $\hat{V}_p^* [1 + V_{fp}(T)]$ | |
| \hat{V}_s^* | = | $9.69 \times 10^{-4} \text{ m}^3/\text{kg}$ (Water) | (Ref. 15) ⁺ |
| $\hat{V}_s^o(190^\circ\text{C})$ | = | $11.429 \times 10^{-4} \text{ m}^3/\text{kg}$ (Water) | (Ref. 18) |
| $V_{fm}(190^\circ\text{C})$ | = | 0.31033 | (Ref. 16) ⁺ |
| $V_{fs}(190^\circ\text{C})$ | = | 0.1795 | (Ref. 15, 18) ⁺ |
| $V_{fp}(T)$ | = | $0.025 + (\alpha_1 - \alpha_g)(T - T_g)$ | (Ref. 19, 21) |
| $\frac{MW_m}{M_{jp}}$ | = | $\frac{2}{3}$ | (Ref. 20) |
| γ | = | 1 | (Ref. 13, 14) |
| $\frac{1}{T_g}$ | = | $\frac{1}{T_{g\infty}} + \left(\frac{K_s}{T_{g\infty}^2} \right) \frac{1}{\bar{\mu}_n}$ | (Ref. 8, 19) |
| $T_{g\infty}$ | = | 350 K | (Ref. 17) |
| α_1 | = | 10^{-3} K^{-1} | (Ref. 17, 21) |
| α_g | = | $4.5 \times 10^{-4} \text{ K}^{-1}$ | (Ref. 17, 21) |
| $K_s/T_{g\infty}^2$ | = | 0.03 K^{-1} | (Ref. 10) |
| K_1 | = | $\exp [(-32.989 - 8024.9/T)/R]$ | (Ref. 22) |
| K_2 | = | $\exp [(3.9846 + 2.4877 \times 10^4/T)/R]$ | (Ref. 22) |
| K_3 | = | $\exp [(-29.06 + 1.6919 \times 10^4/T)/R]$ | (Ref. 22) |
| R | = | 8.314 J/mol-K (Gas Constant) | |

+ estimated values

sphere is assumed to be constant. The mass balance equation for the spherical pellet with diffusion of W in the radial direction can be written as

$$\frac{\partial [W]}{\partial t} = D_w \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial [W]}{\partial r}) + R_w \quad (15)$$

where R_w is the rate of 'generation' of water by chemical reaction (ring opening and polycondensation reactions) and is written as

$$R_w = -k_1 [C_1] [W] + k'_1 [P_1] + k_2 \lambda_o^2 - k'_2 (\lambda_1 - \lambda_o) [W] \quad (16)$$

The complete set of mass balance and moment equations are given in Table IV. The initial conditions can be taken as a uniform distribution of W in the pellet, i. e., $[W] = [W]_o$ for all values of r at $t = 0$. The boundary conditions are easily written and are incorporated in Table IV.

The equations in Table IV can be integrated along with the equations for the apparent rate constants (Table II) for a given set of parameter values, using the finite difference technique (11 grid points) in the radial direction, and Gear's technique.²³ The intrinsic rate constants, $k_{i,o}$, are assumed to be of the following form¹⁰

$$k_{i,o} = \zeta_i \lambda_o ; i=1,2,3 \quad (17)$$

where ζ_1 , ζ_2 and ζ_3 are the parameters (functions of temperature) which are curve fitted in this study. The dependence of $k_{i,o}$ on the concentration, λ_o , of the acid end groups at any location is assumed for the SSP of nylon 6 because it is well known¹² that these groups exert a catalytic effect in melt polymerization of

TABLE IV

FINAL EQUATIONS FOR SOLID STATE POLYMERIZATION OF NYLON 6*

-
1. $\frac{\partial C_1}{\partial t} = -k_1 C_1 W + k'_1 P_1 - k_3 C_1 \lambda_o + k'_3 (\lambda_o - P_1)$
 2. $\frac{\partial P_1}{\partial t} = k_1 C_1 W - k'_1 P_1 - 2k_2 P_1 \lambda_o + 2k'_2 W (\lambda_o - P_1) - k_3 P_1 C_1 + k'_3 P_2$
 3. $\frac{\partial \lambda_o}{\partial t} = k_1 C_1 W - k'_1 P_1 - k_2 \lambda_o^2 + k'_2 W (\lambda_1 - \lambda_o)$
 4. $\frac{\partial \lambda_1}{\partial t} = k_1 C_1 W - k'_1 P_1 + k_3 C_1 \lambda_o - k'_3 (\lambda_o - P_1)$
 5. $\frac{\partial \lambda_2}{\partial t} = k_1 C_1 W - k'_1 P_1 + 2k_2 \lambda_1^2 + \frac{1}{3}k'_2 W (\lambda_1 - \lambda_3) + k_3 C_1 (\lambda_o + 2\lambda_1) + k'_3 (\lambda_o - 2\lambda_1 + P_1)$
 6. $\frac{\partial W}{\partial t} = -k_1 C_1 W - k'_1 P_1 + k_2 \lambda_o^2 - k'_2 W (\lambda_1 - \lambda_o) + D_W \left[\frac{\partial^2 W}{\partial r^2} + \frac{2}{r} \frac{\partial W}{\partial r} \right]$

Initial and Boundary Conditions

7. $t = 0$, values from Table V, independent of radial position
- $r = 0$, $\frac{\partial W}{\partial r} = 0$
- $r = R$, $W = W_s$

Closure Conditions

8. $P_2 = P_1$
 9. $\lambda_3 = \lambda_2 (2\lambda_2 \lambda_o - \lambda_1^2) / (\lambda_1 \lambda_o)$
-

* Brackets, [], for concentrations (local bulk values) omitted for brevity

ϵ -caprolactam, and a similar effect could be present in SSP as well. However, we have also carried out optimal parameter estimation studies assuming constant values of $k_{i,o}$:

$$k_{i,o} = \zeta_i; \quad i = 1, 2, 3 \quad (18)$$

The equilibrium constants for the three reactions in Table I are assumed to be the same as those for melt polymerization.¹² Thus, the intrinsic rate constants for the reverse reactions are given by

$$k'_{i,o} = \frac{k_{i,o}}{K_i} \quad ; \quad i = 1, 2, 3 \quad (19)$$

Since there are no diffusional limitations for the forward and reverse steps of the ring opening reaction, as well as for the backward step of the polyaddition reaction, we have

$$k_1 = k_{1,o} \quad (a)$$

$$k'_1 = k'_{1,o} \quad (b)$$

$$k'_3 = k'_{3,o} \quad (c) \quad (20)$$

The parameters θ_1 , θ_2 and θ_3 have been assumed as curve-fit parameters in this study (along with ζ_i , $i=1,2,3$). It may be added that Achilias and Kiparissides^{13,14} have suggested the use of the theory of excess chain end mobility^{24,25} for estimating these parameters. However, they were forced to introduce a curve fit parameter, $j_{c,o}$, in this theory to obtain good agreement with experimental data on the isothermal polymerization of methyl methacrylate (MMA). Since a curve-fit parameter is necessarily to be used, we decided to use it for the entire group of variables

clubbed together in the θ_i s. A similar procedure was found to be successful in our recent work²⁶ on the modeling of MMA polymerization in semi-batch reactors.

The macro-level diffusivity, D_w , of water through the reaction mass, is an important physical parameter in this work. Strictly speaking, D_w would be expected to decrease with the degree of crystallinity, and so would be a function of the reaction time, since the polymer crystallinity increases as its molecular weight increases. Also, the nonuniform molecular weight profile within the polymerizing particle would imply that the crystallinity would be a function of the distance from the surface of the particle. Thus, the diffusivity is not only time-dependent, but also position-dependent. In this study, however, we have assumed D_w to be constant at an average value. From the available data²⁷ on the diffusivity of water through nylon 6 at a few temperatures, we estimated D_w at the reaction temperature (190°C) to be 10^{-8} m²/hr. Use of this value does not give a good match with experimental results. This value is much too high. On closer scrutiny, it was found that the sample (film) of nylon 6 used to generate the correlation for D_w had a crystallinity of around 30%.²⁸ Gaymans et al.⁵ have not provided any information on the crystallinity of the samples they used for generating experimental results on the SSP of nylon 6. We guess that the crystallinity of their samples (molecular weights around 20,000 - 30,000) could be as high as 75%. This would imply that D_w would be much lower than the value of 10^{-8} m²/hr predicted using the correlation in Ref. 27. Because of this uncertainty, we decided to

treat D_w as a parameter in this simulation, alongwith ζ_i and θ_i ($i=1,2,3$).

In order to obtain the initial conditions to simulate the SSP at 463 K for which some experimental data⁵ are available in the open literature, we first simulated the *previous* stage, namely, melt polymerization¹² of ϵ -caprolactam using $[W]_0 = 0.16$ mol/kg, $[C_1]_0 = 8.8$ mol/kg, $T=513$ K, until a desired value of the number average chain length, μ_n , was obtained. This provided values for all the concentrations and moments at the end of the liquid phase polymerization. These values were taken to be the starting values (at $t=0$) for the SSP, except for the concentrations of ϵ -caprolactam and water. $[C_1]$ and $[W]$ at $t=0$ for the SSP, were taken as zero to duplicate their extraction out of the pellets produced experimentally by Gaymans et al.⁵ from melt polymerized nylon 6. The starting concentrations used for simulation of SSP of nylon 6 are listed in Table V for their different experimental runs⁵ (corresponding to different values of the initial number average chain length, $\mu_{n,0}$). It must be emphasized that the complete details on the conditions used by Gaymans et al.⁵ for producing the initial samples for SSP are not available in their paper, and this is why we have tried to generate initial values using the above technique, trying to match the only variable ($\mu_{n,0}$) whose value is mentioned.

An optimal parameter estimation computer program has been used to obtain the best-fit values of the seven parameters (ζ_i , θ_i ; $i=1,2,3$; D_w) using the isothermal experimental data on nylon 6 SSP. The Box-complex algorithm^{29,30} has been used for this

TABLE V
STARTING *CONCENTRATIONS FOR SSP*

| INITIAL CONCENTRATIONS (mol/kg) FOR DIFFERENT RUNS OF SSP | | | | | |
|---|------------------|---------|---------|---------|---------|
| | $\mu_{n,o} = 22$ | 35 | 55 | 72 | 95 |
| $[C_1]$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $[P_1]$ | 0.00075 | 0.00068 | 0.0006 | 0.00055 | 0.00049 |
| λ_o | 0.03122 | 0.04331 | 0.05244 | 0.05456 | 0.05367 |
| λ_1 | 0.69388 | 1.53307 | 2.93076 | 3.95800 | 5.13901 |
| λ_2 | 22.10 | 80.43 | 260.10 | 485.20 | 902.20 |
| $[W]$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

* Initial conditions in liquid phase polymerization to obtain the above values :

| | |
|-----------------|---------------|
| Temperature | = 513.0 K |
| $[C_1]_o$ | = 8.8 mol/kg |
| $[P_1]_o$ | = 0.0 mol/kg |
| $\lambda_{o,o}$ | = 0.0 mol/kg |
| $\lambda_{1,o}$ | = 0.0 mol/kg |
| $\lambda_{2,o}$ | = 0.0 mol/kg |
| $[W]_o$ | = 0.16 mol/kg |

purpose. The objective function that has been optimized in the program is

$$F(\zeta_i, \theta_i; i=1,2,3; \mathbb{D}_w) = \text{Max} \left[- \sum_{j=1}^N \frac{\text{abs}\{\mu_{n,\text{exptl}}(j) - \mu_{n,\text{theor}}(j)\}}{\mu_{n,\text{theor}}(j)} \right] \quad (21)$$

where N is the total number of experimental data points for all $\mu_{n,o}$ reported⁵, and the subscripts, *exptl* and *theor*, indicate the experimental and theoretical values of μ_n .

CHAPTER 3

RESULTS AND DISCUSSION

The computer program was tested for several special cases to ensure that it was free of errors. The number of finite difference grid points was increased from 11 to 21 and the results on $\bar{\mu}_n(t)$ were found to be essentially the same (within 1% at $t = 24$ hr). Similarly, the results were unaffected when the parameter, TOL, in the NAG library subroutine, D02EBF, was decreased from 10^{-4} to 10^{-8} . After these tests, the Box complex method was used to obtain the best-fit values of the seven curve-fit parameters, using both Eqns. (17) and (18) for $k_{i,o}$. Table VI gives the final values of these parameters for both these cases.

Simulation results for the five different experimental runs⁵ (corresponding to different values of the initial number average chain length, $\mu_{n,o}$) using the best-fit values of the parameters (Table VI) are shown by the solid curves in Figs. 2 and 3 for the case when the intrinsic rate constants, $k_{i,o}$, are proportional to the zeroth moment (Eqn. 17). The experimental data points are also shown in these diagrams. Excellent agreement between theory and experimental data is observed. The dotted curves in these diagrams show the theoretical results using the best-fit values of the parameters with $k_{i,o} = \zeta_i$ (Eqn. 18). The final values of the objective function, F (in Eqn. 21), for these two choices of $k_{i,o}$ are -2.53 and -1.89 respectively. Based on these values as well as on the agreement between the theoretical curves and experimental data in Figs. 2 and 3 it is difficult to say which of these two correlations for the intrinsic rate constants is superior, and we

TABLE VI
BEST-FIT VALUES OF THE PARAMETERS USED FOR THE
SIMULATION OF SSP OF NYLON 6 AT 190 °C

| <u>Optimal Parameters</u> | for $k_{i,o} = \zeta_i \lambda_o$ | for $k_{i,o} = \zeta_i$ |
|--------------------------------------|---|----------------------------|
| θ_1 (hr) | 4.626×10^{-4} | 3.339×10^{-4} |
| θ_2 (hr) | 406.557 | 486.251 |
| θ_3 (hr) | 6476.71 | 2586.66 |
| ζ_1 | 49.183 (kg ² /mol ² -hr) | 0.3247 (kg/mol-hr) |
| ζ_2 | 14996.9 (kg ² /mol ² -hr) | 405.7 (kg/mol-hr) |
| ζ_3 | 360.59 (kg ² /mol ² -hr) | 7.7116 (kg/mol-hr) |
| D_w (m ² /hr) | 3.366×10^{-11} | 1.065×10^{-11} |
| <u>Other Parameters / Conditions</u> | | |
| R = | 2×10^{-4} m | |
| [W] _s = | 0.00001 mol/kg | |
| T = | 463 K | |

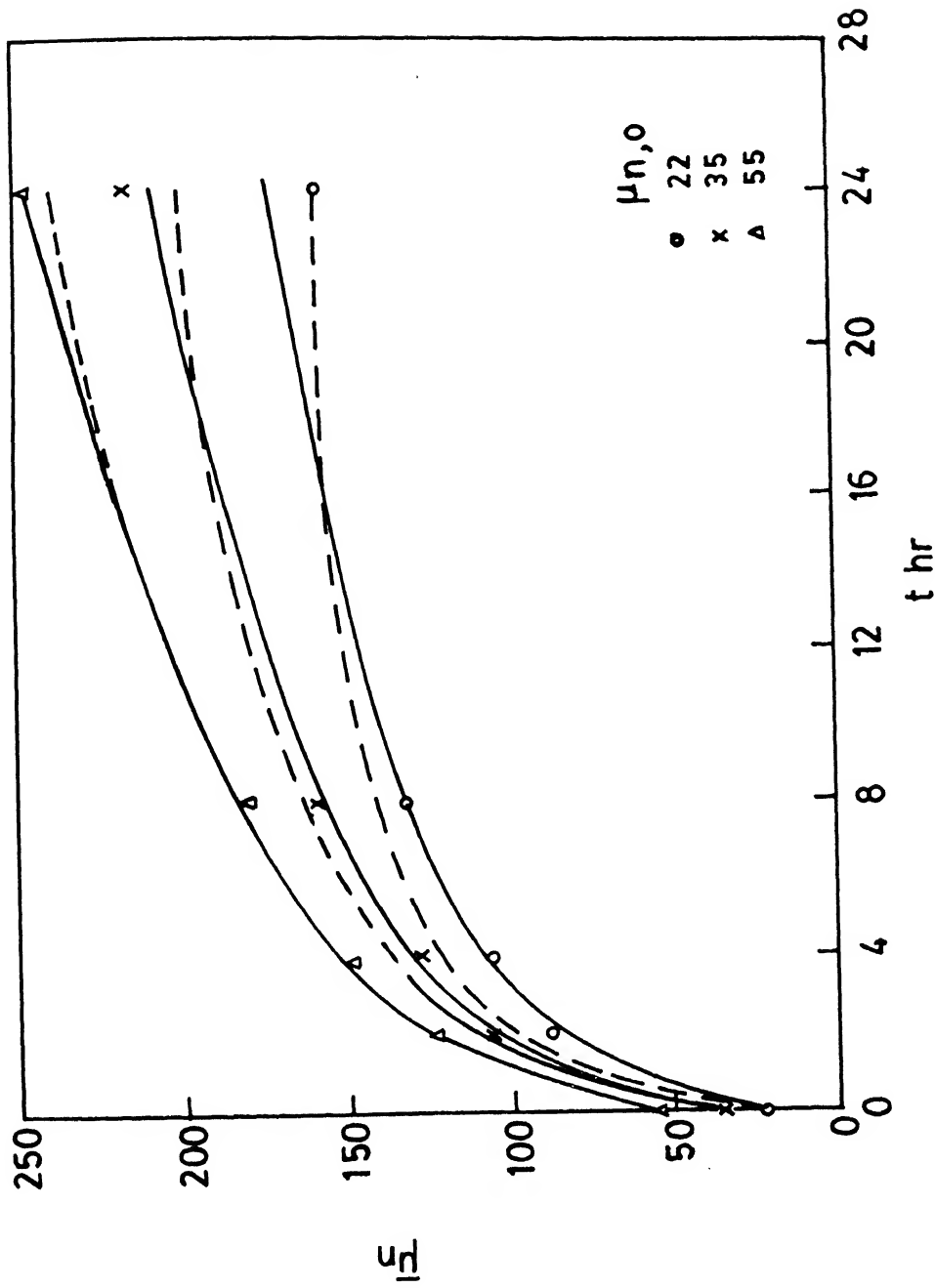


FIG. 2

Comparison of simulation results with experimental data at 190 °C for $\mu_{n,0} = 22, 35, 55$. Reference values of the parameters (Table VI) used. Eqns. 17 ($k_{i,0} = \zeta_i \lambda_o$) and 18 ($k_{i,0} = \zeta_i$) used for $k_{i,0}$.

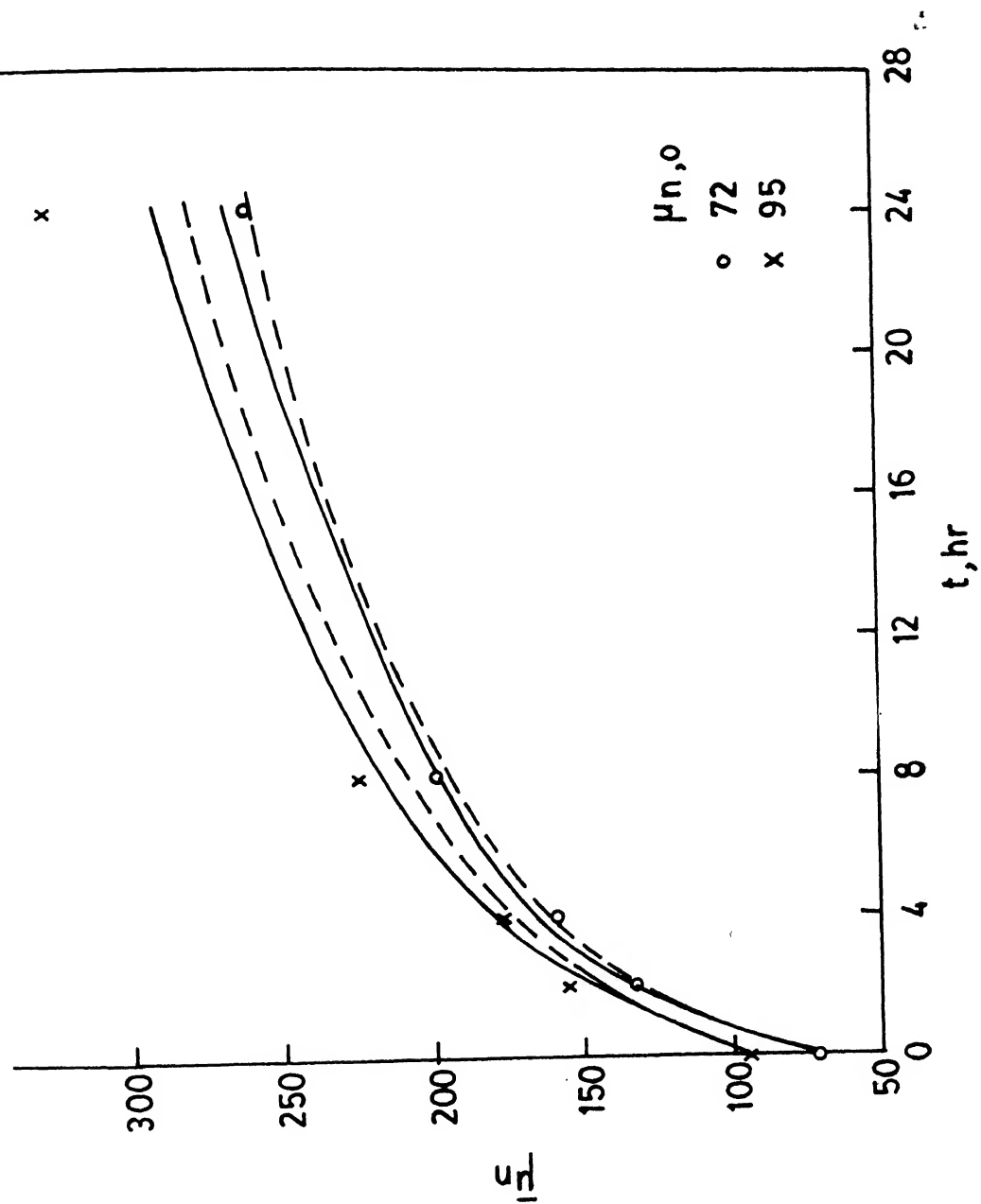


FIG. 3

Comparison of simulation results with experimental data at 190°C for $\mu_{n,0} = 72, 95$. Reference values of the parameters (Table VI) used. Same notation as in

Fig. 2.

have arbitrarily selected the form in Eqn. (17) for further study in this work. A comparison of the present results with those from our earlier work¹⁰ which used the Fujita-Doolittle theory for the micro-level diffusivity reveals that the agreement of the present theory is better. In addition, the present theory is more elegant since the parameters of the Vrentas-Duda theory are obtained by independent measurements on nonreacting systems.

The sensitivity of the results to changes in the parameters is now studied (using Eqn. 17 for $k_{i,o}$). The value of $\mu_{n,o}$ of 55 is selected for this purpose. Figs. 4-8 show the effects of varying the seven different parameters one by one, about their best-fit values, keeping the other parameters at their reference (Table VI) values. The effect of θ_1 on $\bar{\mu}_n$ is shown in Fig. 4. Decreasing θ_1 speeds up the reaction, particularly in the initial stages, and increases the final values of $\bar{\mu}_n$. Lower values of θ_1 signify higher values of the micro-level diffusivity of the polymer molecule in the reaction mass. This leads to an increase in the forward rate constant of the polycondensation reaction (other quantities remaining unchanged) and hence in the values of $\bar{\mu}_n$. Very high values of θ_1 (>0.01 hr) suppress the forward step of the polycondensation reaction so much that depolymerization takes place during the early stages. It is interesting to note that this figure illustrates how the presence of diffusional limitations (non zero values of θ_1) reduces the value of $\bar{\mu}_n$ significantly.

The effect of varying the parameter, θ_2 , on the progress of reaction is shown in Fig. 5. Increased values of θ_2 lead to a

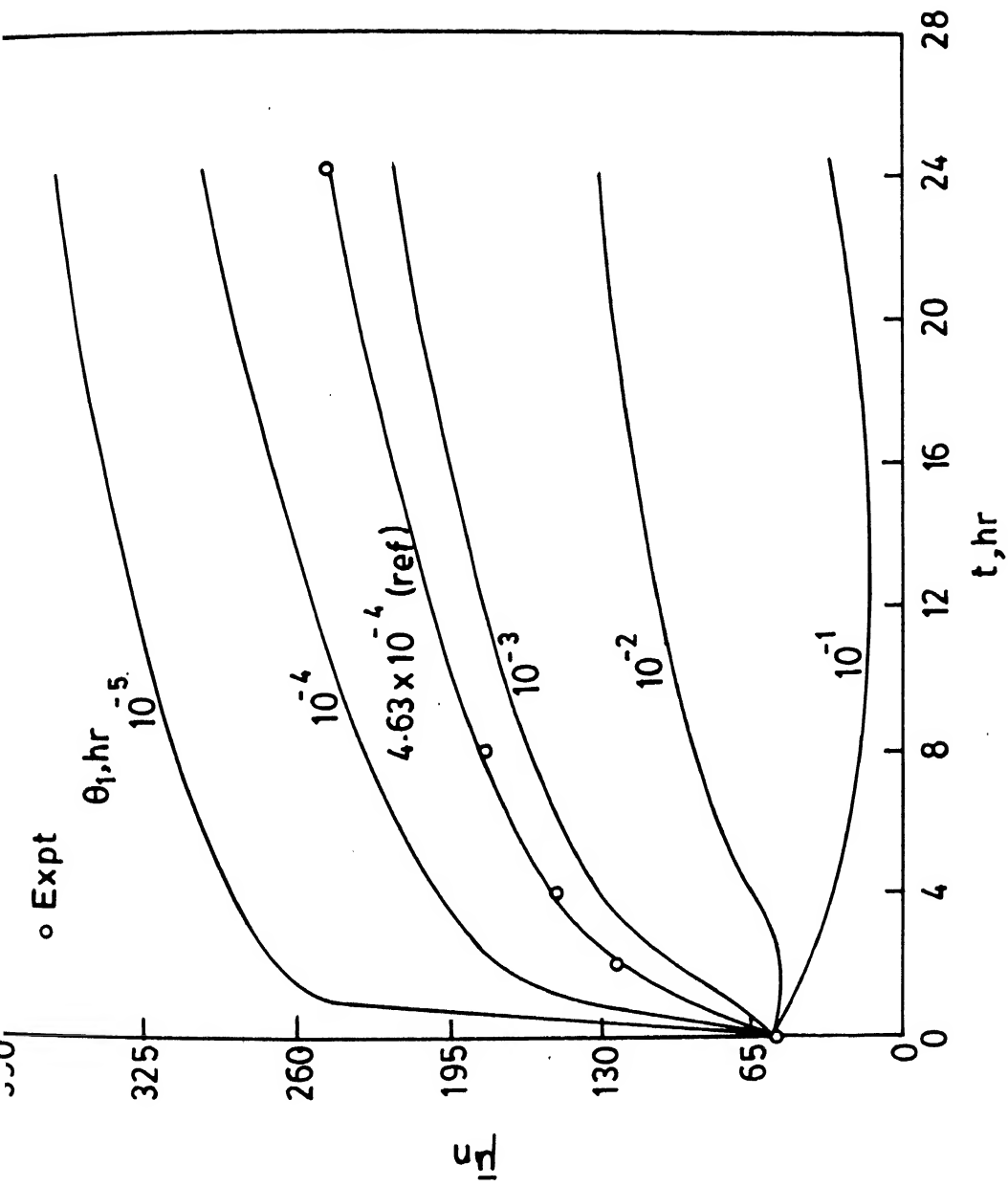


FIG. 4

Effect of θ_1 on $\bar{\mu}_n$. $T = 190^\circ\text{C}$, $\mu_{n,0} = 55$, $k_{i,0} = \zeta_i \lambda_0$ (Eqn. 17). Reference values of the other parameters (Table VI) used.

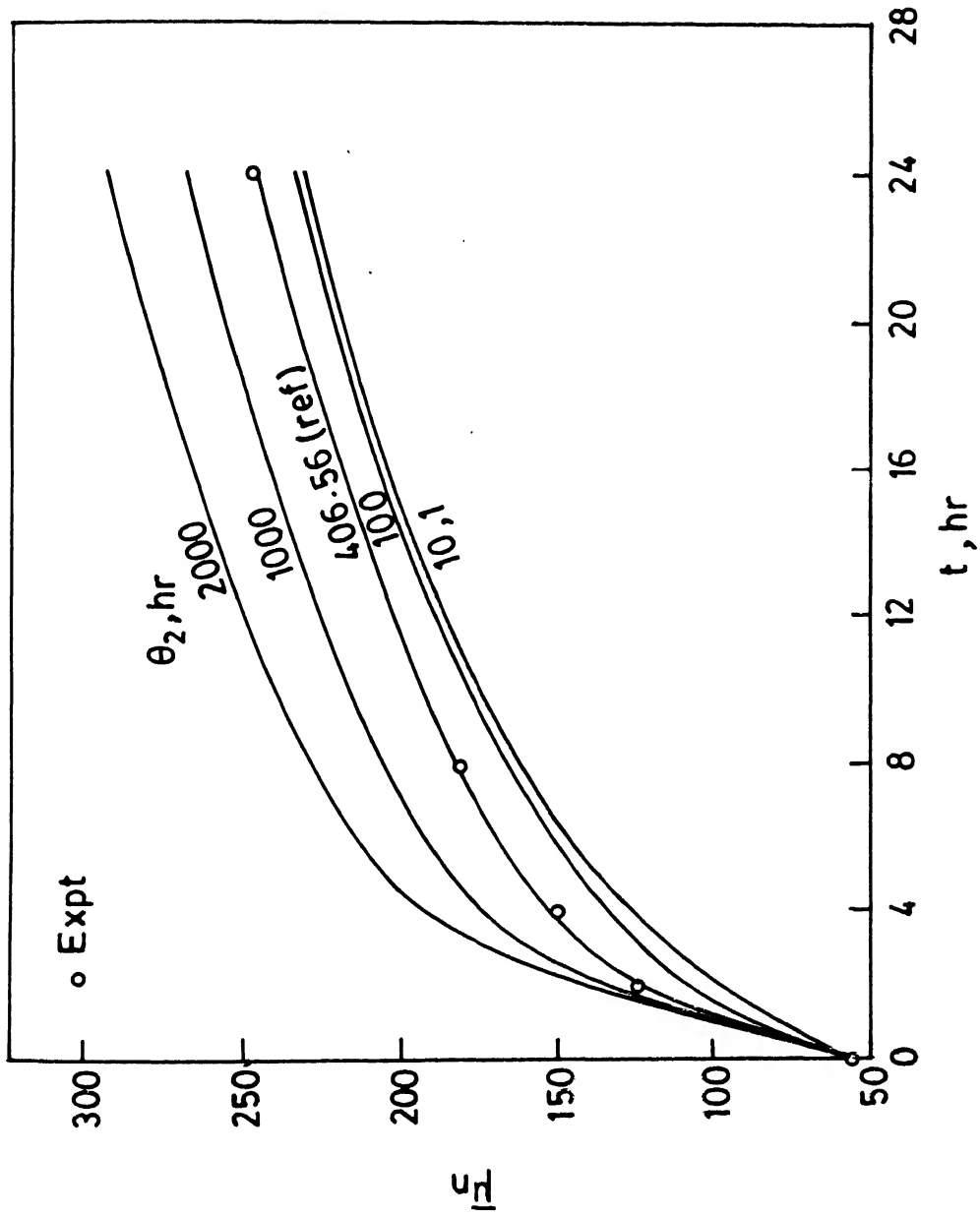


FIG. 5

Effect of θ_2 on $\bar{\mu}_n$. Reference values of the other parameters used.

decrease in the micro-level diffusivity of the water molecule and a corresponding decrease in the reverse rate constant, k'_2 (other quantities remaining unchanged). This means that the polycondensation reaction is forced towards the right, leading to higher values of $\bar{\mu}_n$. The sensitivity of the results to θ_2 is less than that towards θ_1 .

The results are found to be relatively insensitive to the value of θ_3 . Decreasing θ_3 from the reference values of 6476.7 hr to 100 hr decreases the value of $\bar{\mu}_n$ at $t = 24$ hr only from about 246.1 to 220.65. A similar insensitivity of $\bar{\mu}_n$ is observed to the value of ζ_1 (between 10 hr to 1000 hr). In fact, use of $\zeta_1 = 0$ (which makes $k'_{1,0} = 0$) also gave the same $\bar{\mu}_n$ history. This means that the ring opening reaction in Table I can easily be omitted. Indeed this reaction was omitted in our previous study,¹⁰ based on intuitive reasoning. It may be emphasized that we cannot omit the polyaddition reaction from the kinetic scheme even though we get the same results with $\theta_3 \rightarrow \infty$ (i.e., $k_3 = 0$). This is because the rate constant, $k'_{3,0}$ of the reverse step does not become zero as $\theta_3 \rightarrow \infty$. Indeed, this reverse reaction is responsible for increasing the value of $[C_1]$ in the polymer particle from its initial value of zero, as the polymerization progresses.

Fig. 6 shows the effect of varying ζ_2 . Increasing the value of ζ_2 beyond the reference value of 14996.9 to 22000 $\text{kg}^2 \text{mol}^{-1} \text{hr}^{-1}$ does not affect the progress of the reaction much, but lowering the values of ζ_2 does lead to considerable effect on the progress of the reaction. In fact, it is observed that as ζ_2 is increased from a low value of 100 to 22000 $\text{kg}^2 \text{mol}^{-1} \text{hr}^{-1}$, $\bar{\mu}_n(t)$ goes up with

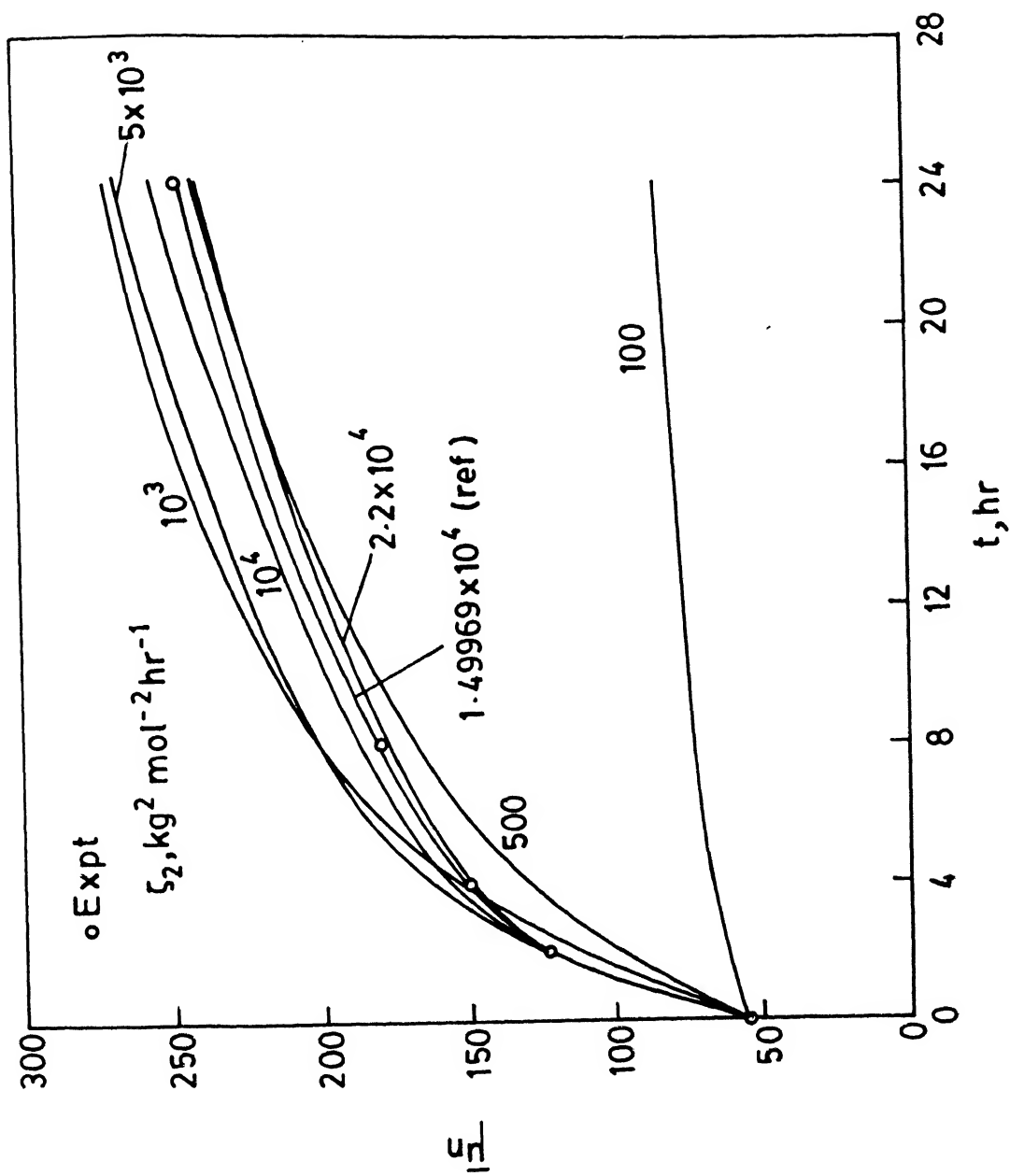


FIG. 6

Effect of ζ_2 on $\bar{\mu}_n$. Reference values of other parameters used.

time more rapidly near $t \approx 0$ (due to higher rates of the forward step in the polycondensation reaction), but flattens out at larger values of t to give lower final values of $\bar{\mu}_n$. The latter is because diffusional limitations become more pronounced and occur earlier for higher values of ζ_2 . Fig. 6, thus, illustrates the effect of competing physical phenomena. A similar phenomenon is observed in Fig. 7, where ζ_3 is varied (note ζ_3 going from 500 to $1500 \text{ kg}^2 \text{mol}^{-2} \text{hr}^{-1}$). The curve for $\zeta_3=0$ (which makes $k'_{3,o} = 0$ also) in Fig. 7 shows what happens when we omit the polyaddition reaction from our kinetic scheme. This behavior is to be contrasted with the curve for $\xi_2 = 100 \text{ kg}^2 \text{mol}^{-1} \text{hr}^{-1}$ in Fig. 6. The importance of the polycondensation reaction over the polyaddition reaction is quite evident.

The macro-level diffusivity of water through the reaction mass, D_w , (or the degree of crystallinity) plays an important role in SSP as shown in Fig. 8. For values of D_w above about $10^{-10} \text{ m}^2/\text{hr}$ (corresponding to higher fractions of amorphous material) the values of $\bar{\mu}_n$ are higher. Values of D_w below the reference value of $3.37 \times 10^{-11} \text{ m}^2/\text{hr}$ lead to lower $\bar{\mu}_n$. The results become relatively insensitive to D_w below a value of about $10^{-13} \text{ m}^2/\text{hr}$. For such cases, the water produced by chemical reaction does not diffuse out much, and the reaction is driven towards final values of $\bar{\mu}_n$ controlled by equilibrium alone. One need not solve partial differential equations for such cases. It should be emphasized that the degree of crystallinity would affect not only D_w but also the values of θ_1 and its influence would be more than depicted by Fig. 8.

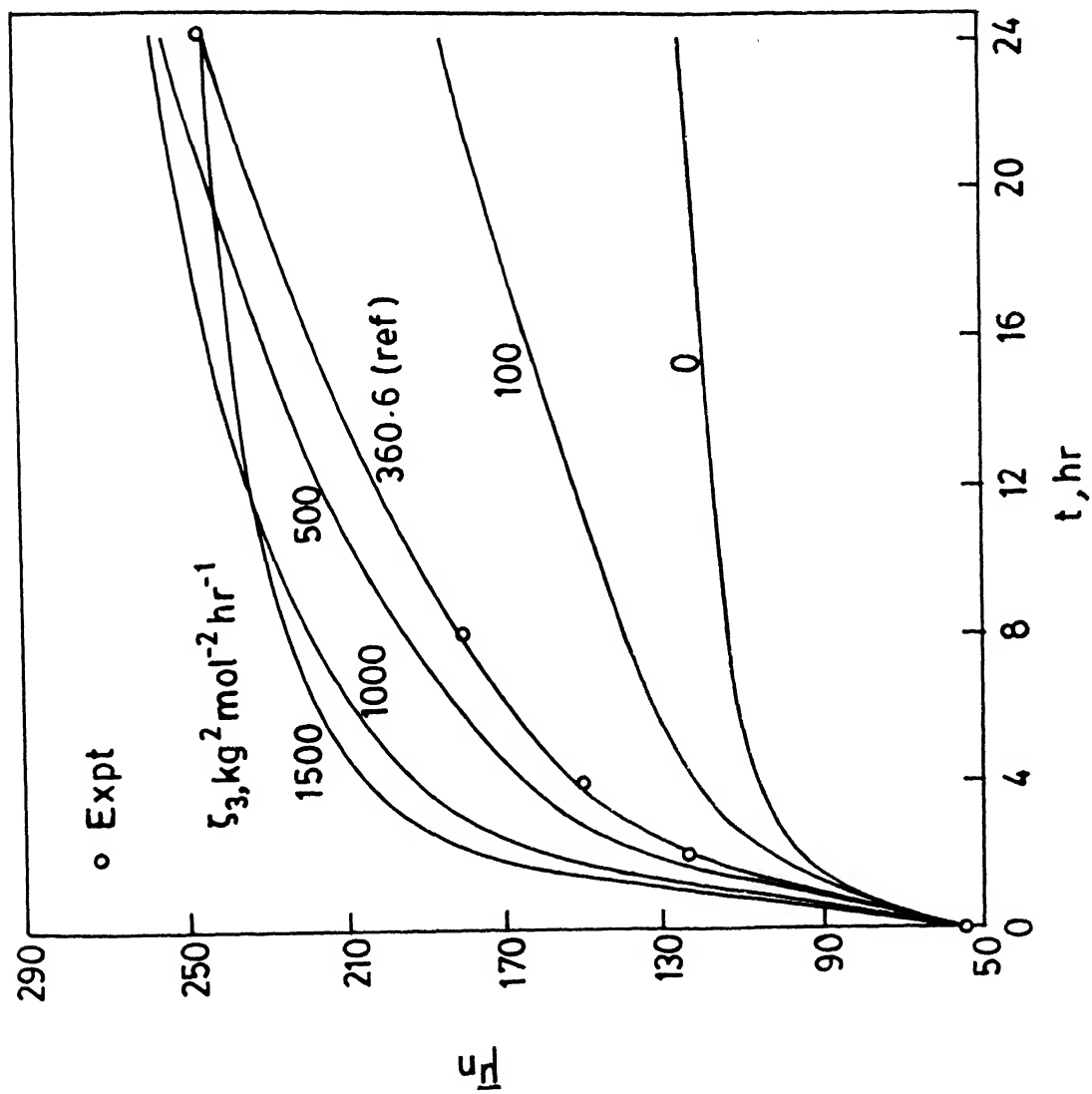


FIG. 7

Effect of ζ_3 on $\bar{\mu}_n$. Reference values of other parameters used.

The effect of varying some important operating conditions has also been studied. The increase in $\bar{\mu}_n$ is found to be relatively insensitive to the surface concentration of water, $[W]_s$, when its value is below about 10^{-3} mol/kg (see Fig. 9). For values of $[W]_s$ above about 10^{-3} mol/kg, however, the results are found to be sensitive to its value. This implies, technologically, that one needs to reduce the concentration of water in the vapor phase surrounding the nylon 6 to a threshold value, and not much is achieved in going below this level. It is interesting to note that the difference in $[W]_s$ shows up only after about 6 hr of SSP. The radius, R , of the nylon 6 powder (assumed spherical) is within the range of 0.2 mm - 0.5 mm, as reported by Gaymans et al⁵. The reference value used in this study is 0.2 mm. It is found that (see Fig. 10) decrease in the value of R from 0.5 to 0.2 mm causes an increase in the final value of $\bar{\mu}_n$ of about 40 to 50. It is interesting to note that much more is achieved by reducing the pellet size from 0.3 mm to 0.2 mm than from 0.5 mm to 0.4 mm. This is because the major part of polymerization occurs in a thin shell near the periphery (see later). Figs. 9 and 10 quantify the effect of two of the most important operating conditions.

We also carried out a simulation run for the case when the monomer and water are not leached out of the polymer particles before the SSP, using the reference values of all the parameters (for the case when $k_{i,o} = \zeta_i \lambda_o$). The values of $[C_1]_o$ and $[W]_o$ are 5.869 and 0.09683 mol/kg respectively (for $\mu_{n,o} = 55$). A sudden jump in $\bar{\mu}_n$ is observed near $t \sim 0$, but the values for t larger than about 9 hr, are much lower (213.7 as compared to 246.1 at $t =$

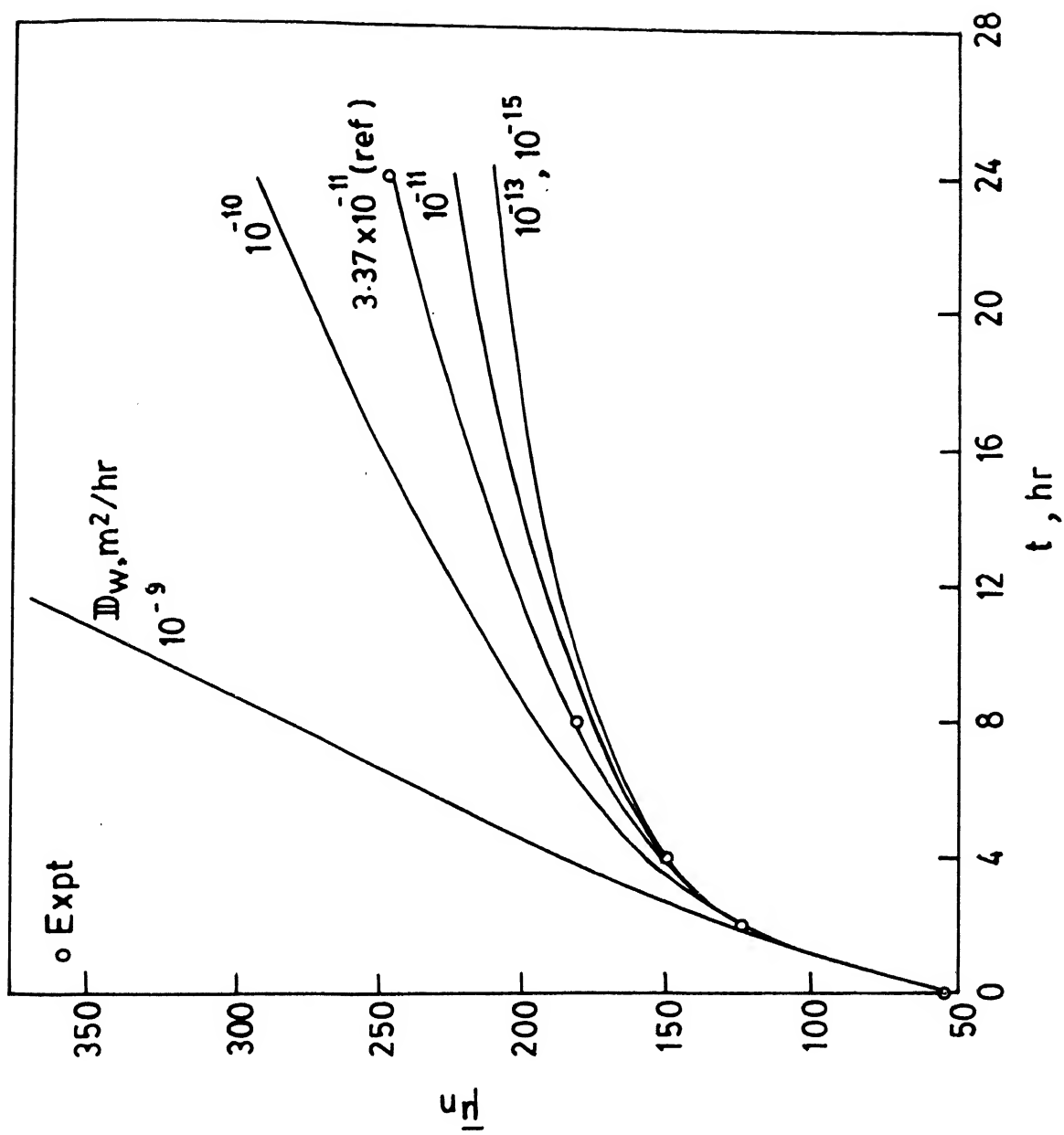


FIG. 8

Effect of the D_w on $\bar{\mu}_w$. Reference values of other parameters used.

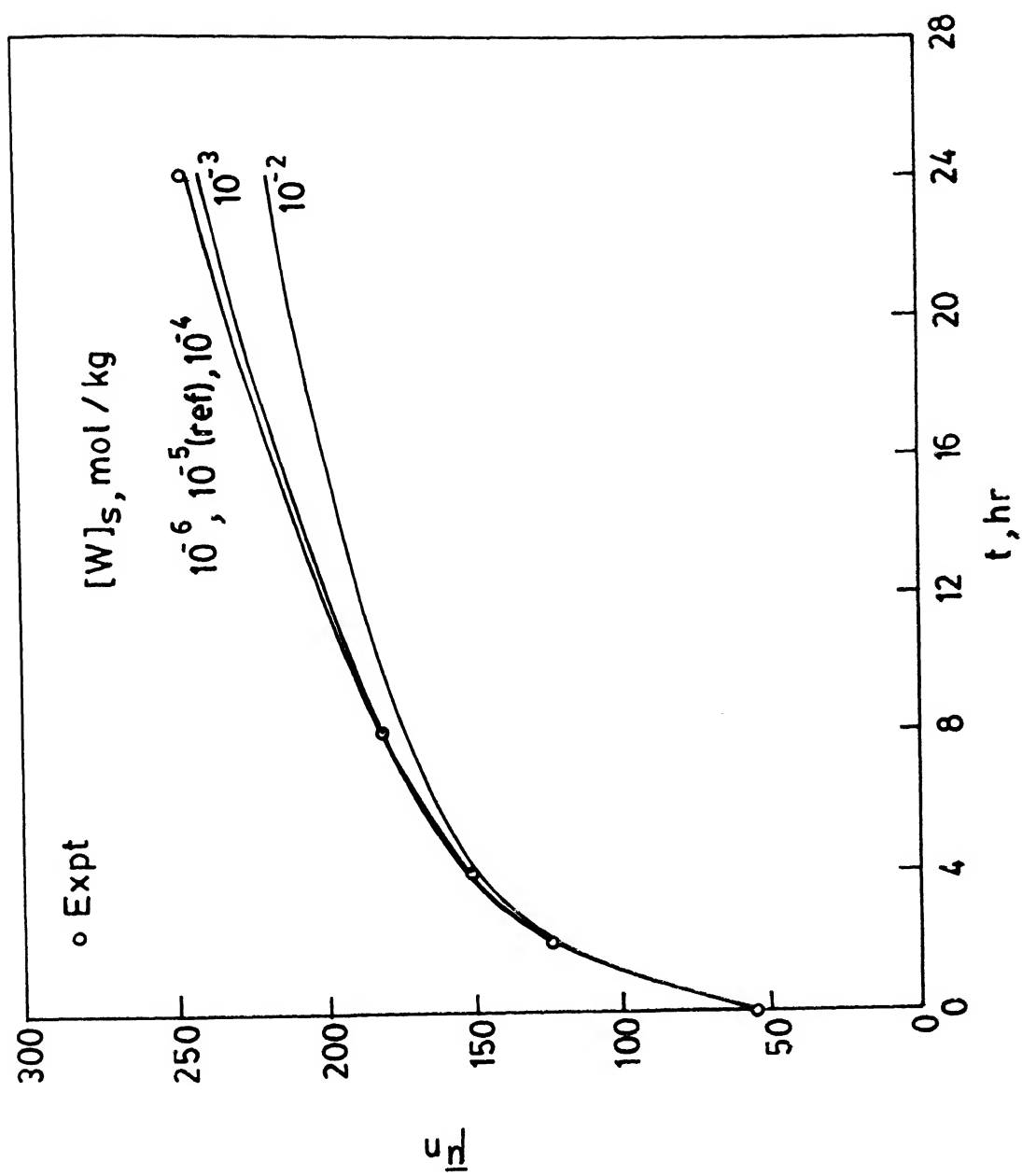


FIG. 9

Variation of $\bar{\mu}_n$ with time as $[W]_s$ is changed.
Reference values of all parameters used.

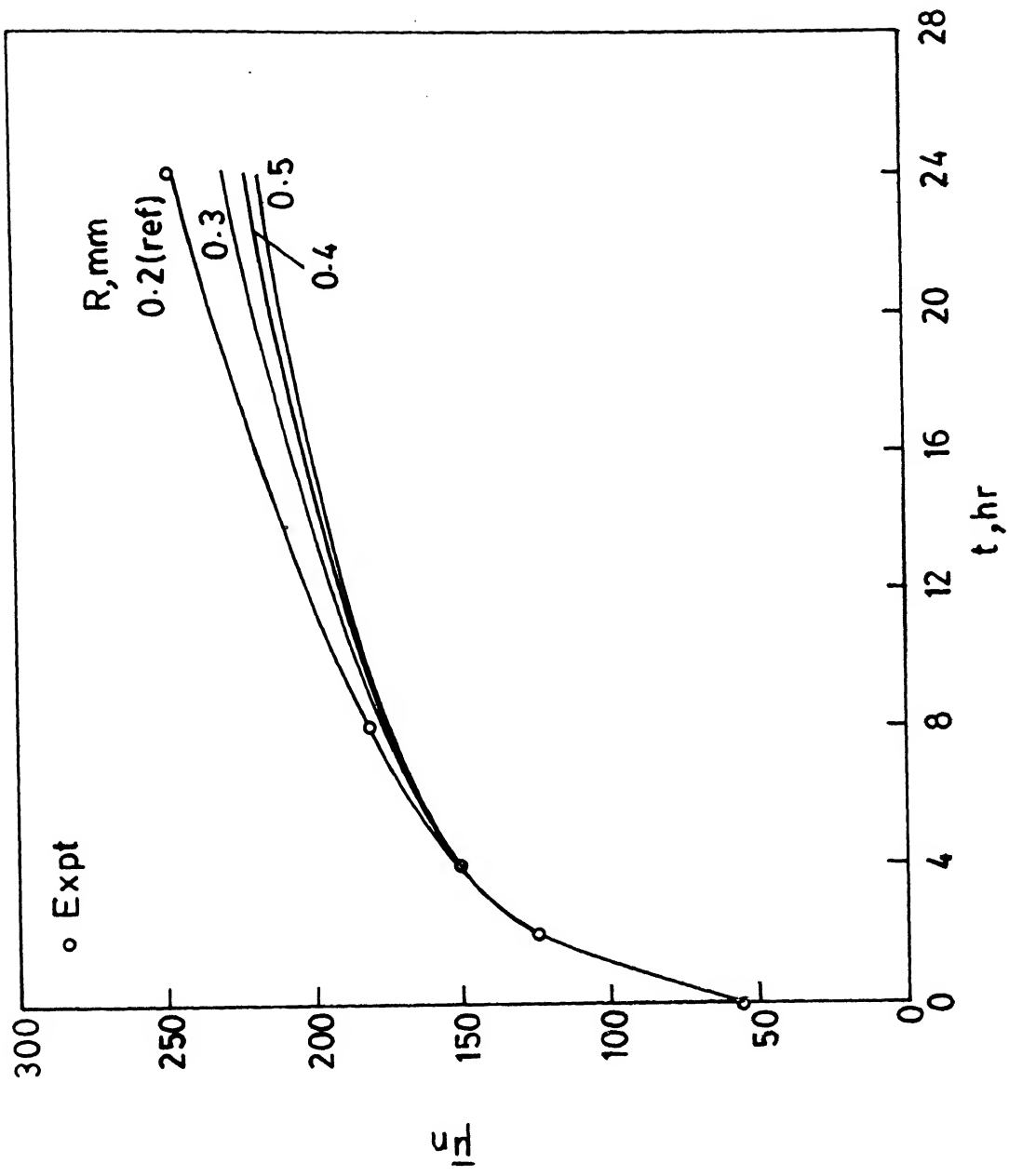


FIG. 10

Variation of $\bar{\mu}_n$ with time as R is changed. All other conditions are the same as for the reference run.

24 hr). The sharp initial rise in $\bar{\mu}_n$ is due to high initial concentration of the monomer which drives the polyaddition reaction to the right-hand side. This effect dominates over the effect of higher $[W]$ values, which drives the polycondensation reaction to the left (depolymerization). The lower final values of $\bar{\mu}_n$ emphasizes the need for carrying out leaching before SSP.

Fig. 11 shows the results for SSP of nylon 6 with a single intermediate remelting (homogenizing) of the polymer particles. Reference values (Table VI) of the parameters have been used. Points p, q, r and s in this figure show that if the total time for SSP (i.e., before and after remelting) is the same (24 hr), higher $\bar{\mu}_n$ product can be obtained by intermediate and early homogenization. The results are not as dramatic if the homogenization occurs late. This trend is qualitatively similar to the experimental observations of Gaymans et al.⁵ The higher values of $\bar{\mu}_n$ result from lower (uniform) values of $[W]$ in the inside of the polymer particles, resulting from homogenization.

Fig. 12 shows the water concentration profiles at different values of t (in absence of remelting) for the reference conditions. It is observed that the water concentration builds up in the interior of the polymer particle (due to generation) quite early during the reaction. However, macro-level diffusion of W leads to a relatively sharp decrease in $[W]$ in the outer shell of the particle, $0.16 \text{ mm} < r < 0.2 \text{ mm}$. The lower water concentrations in the outer shell are associated with significantly higher values of μ_n in this region, as shown in Fig. 13. This also explains why remelting of the polymer particle leads

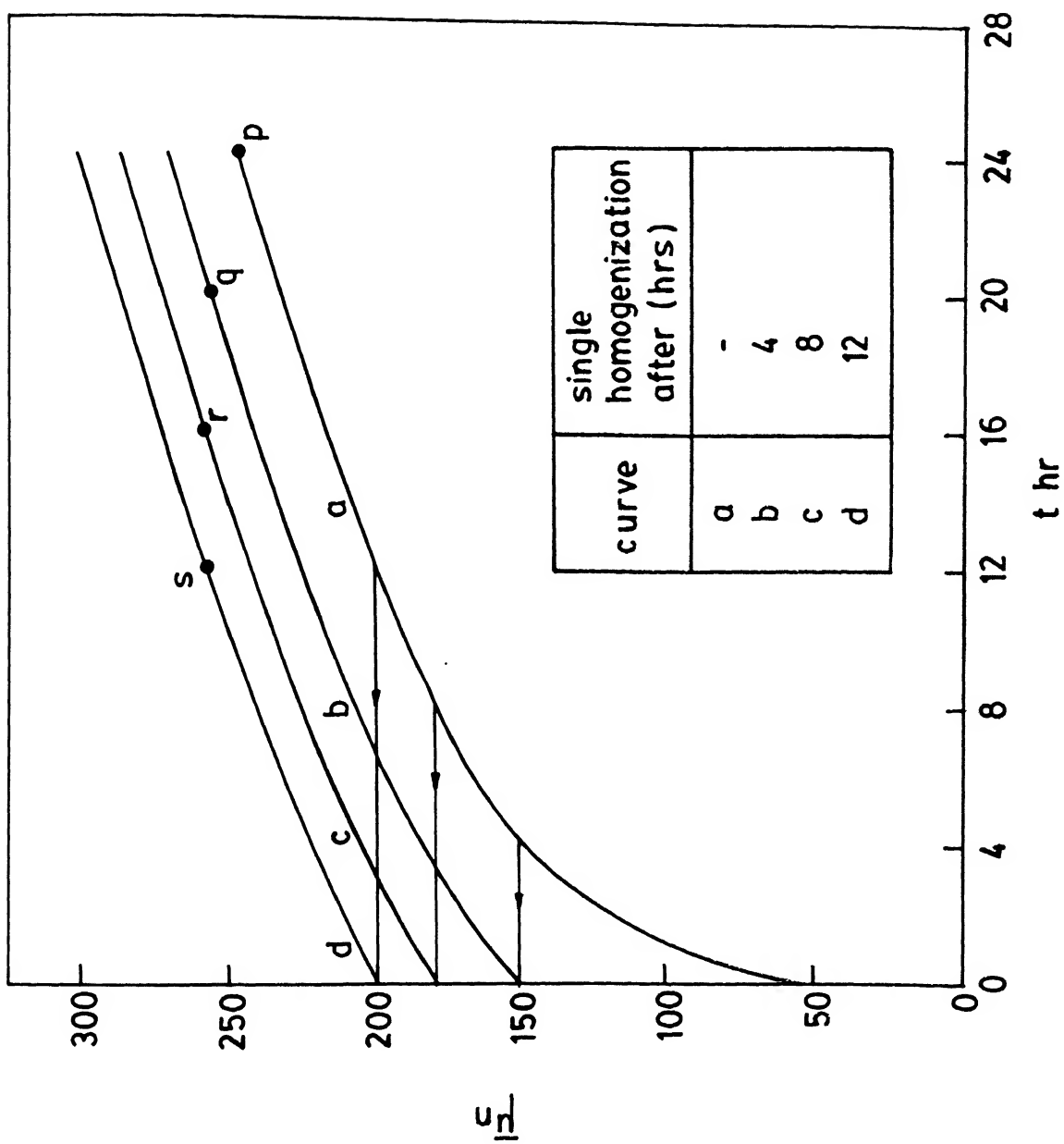


FIG. 11

Comparison of the results for SSP with single intermediate homogenization (remelting) of polymer

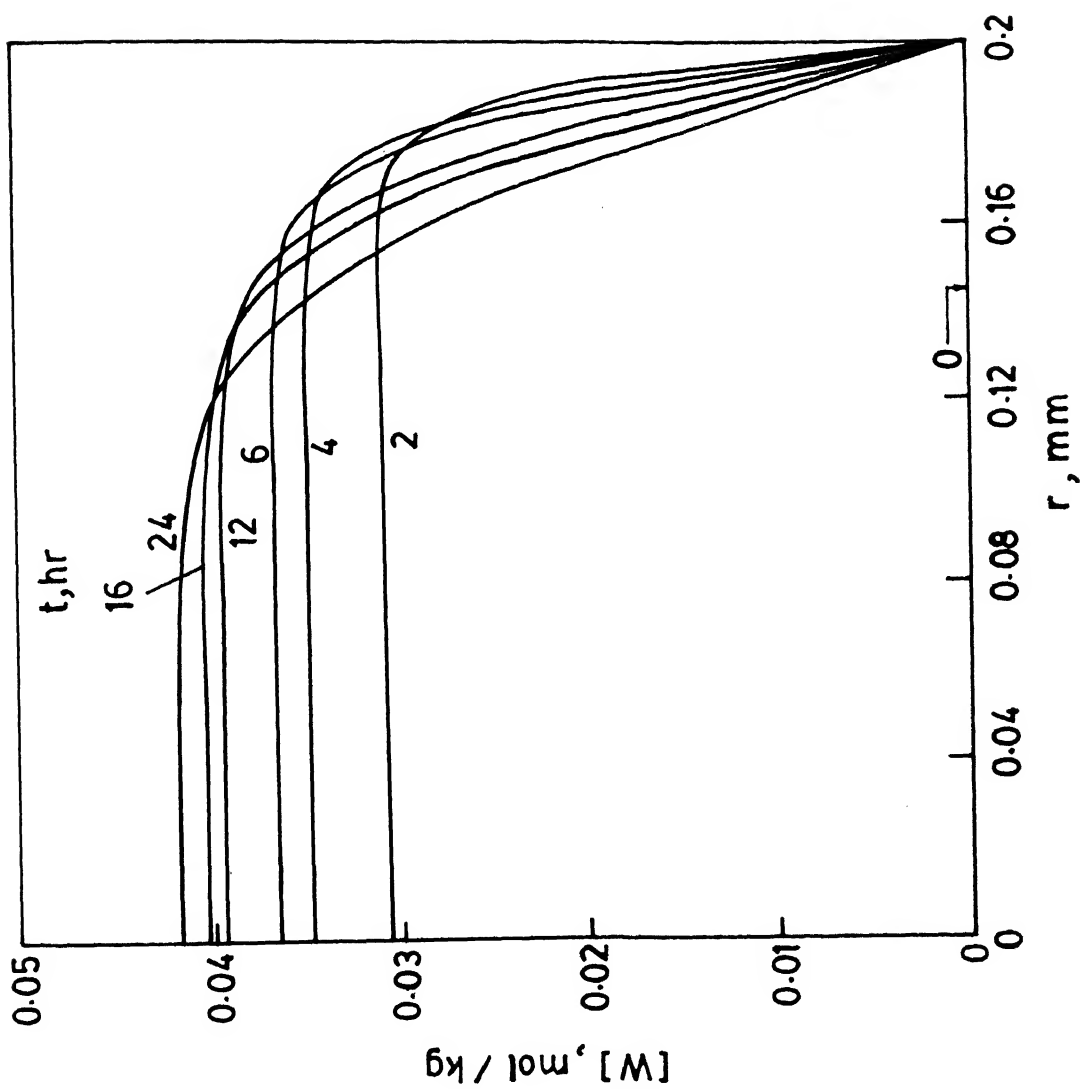


FIG. 12

Radial profile of $[W]$ at different times for the reference run (with $\mu_{n,o} = 55$ and with Eqn. 17).

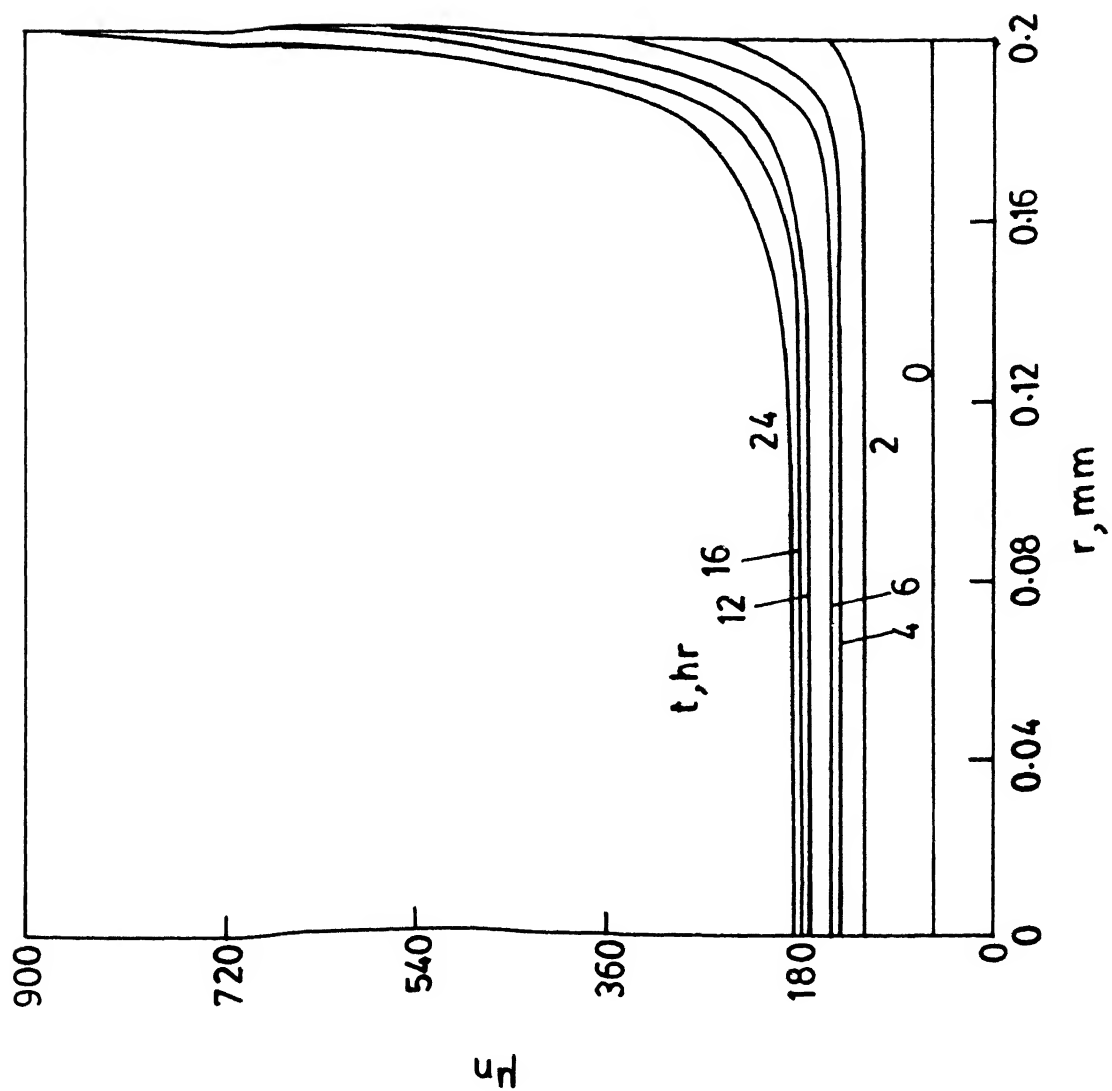


FIG. 13

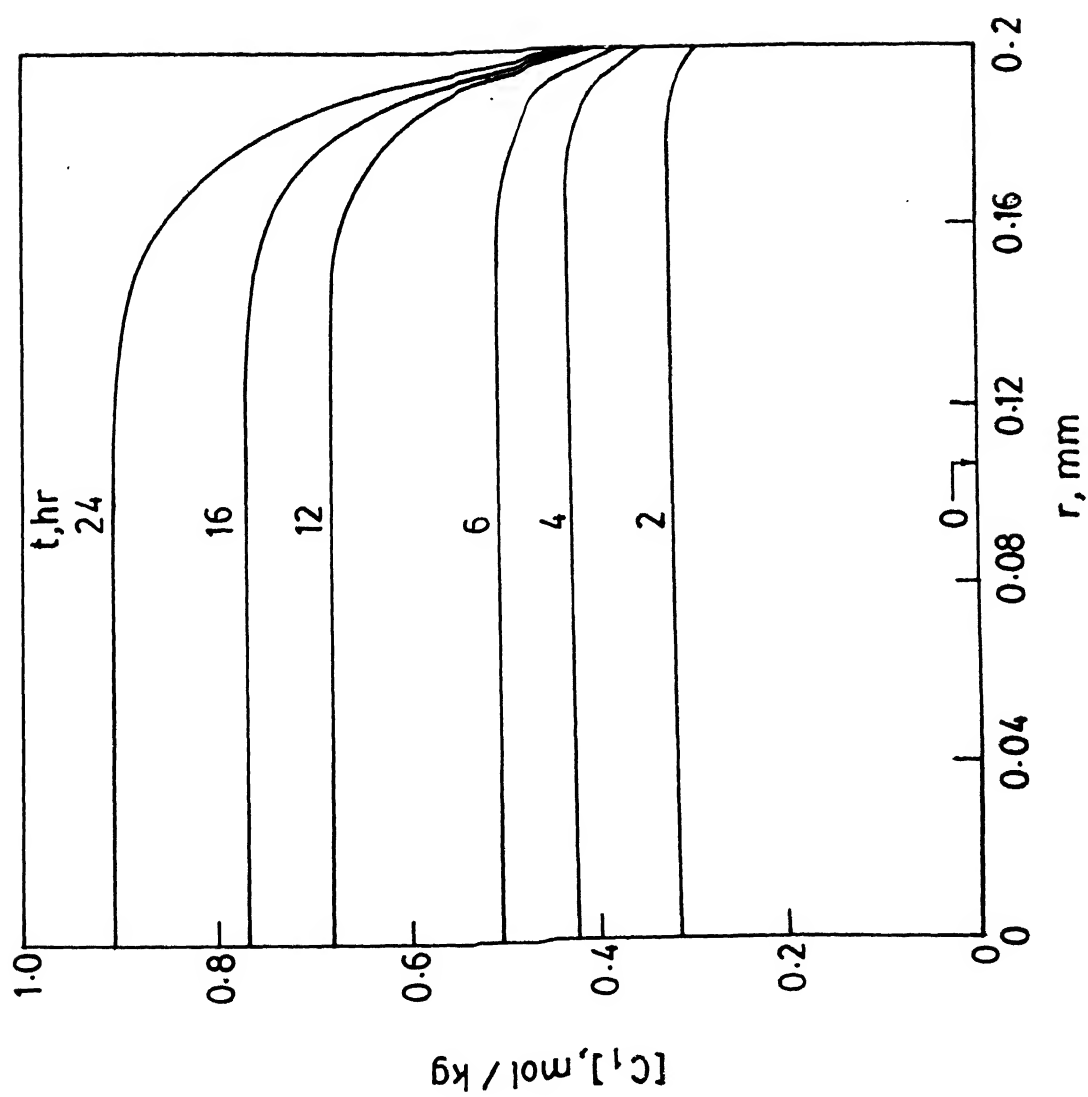
Radial profile of μ_n at different times for the reference run.

to higher $\bar{\mu}_n$ product, as shown in Fig. 11, since intermediate homogenization removes the spatial gradient, and reduces the averaged value of $[W]$ in the inner regions. It may be added that these profiles have been generated using 21 finite-difference grid points because of the relatively sharp gradients near $r = R$.

Fig. 14 shows how the monomer concentration increases from its initial value of zero as the reaction progresses. Since it has already been found that the ring-opening reaction is unimportant, this means that the reverse step of the polyaddition reaction is responsible for the generation of the monomer. The reason why $[C_1]$ is lower near the outer fringes of the polymer particle is because the total concentration of polymer (i.e., $\sum [P_n]$) molecules is lower there (associated with the higher values of μ_n).

The variation of μ_n with location would be expected to lead to values of the spatially averaged (mixed) polydispersity index, \bar{Q} , above the normal value of 2.0. Fig. 15 shows how \bar{Q} increases with time for the five cases studied. \bar{Q} is found to rise rapidly at first, and then increases more gradually to values above 2.0. The initial values of \bar{Q} are not 2.0 for the low $\mu_{n,0}$ cases due to low conversions at the beginning.

It is interesting to note (see Fig. 16) that the apparent rate constant, k_2 , increases with time, rather than decreases. This diagram shows k_2 at two positions, one near the center of the spherical polymer particle (at $r = 0.04$ mm), and the other near the periphery (at $r = 0.18$ mm) for the reference run (with $\mu_{n,0} = 55$). The value of the intrinsic rate constant, $k_{2,0}$, is found to

**FIG. 14**

Radial profile of $[C_1]$ at different times for the reference run.

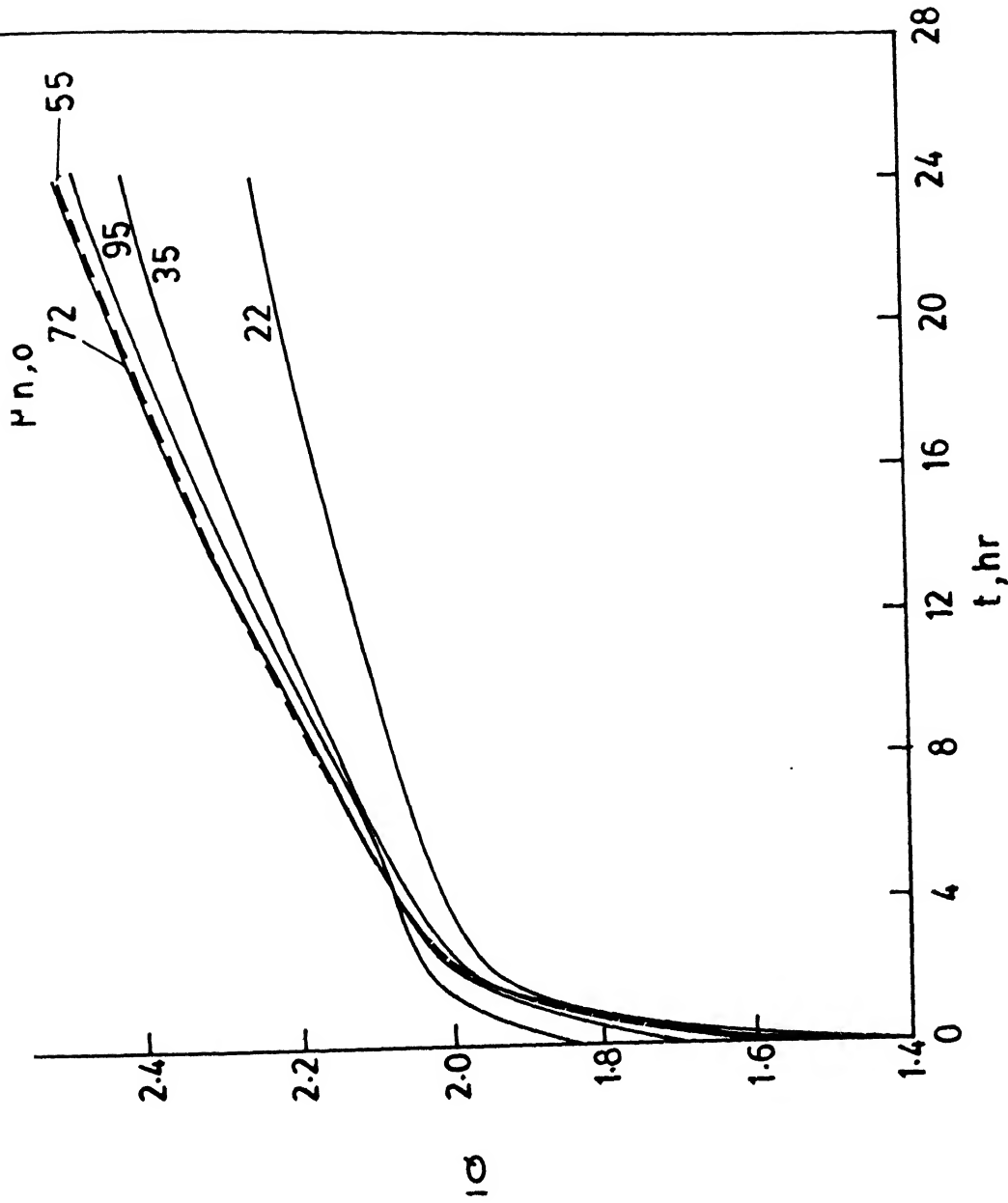


FIG. 15

Variation of average polydispersity index, \bar{Q} , with time for $\mu_{n,0} = 22, 55, 72, 95$. Eqn. (17) used for $k_{i,0}$. Reference values of all parameters used.

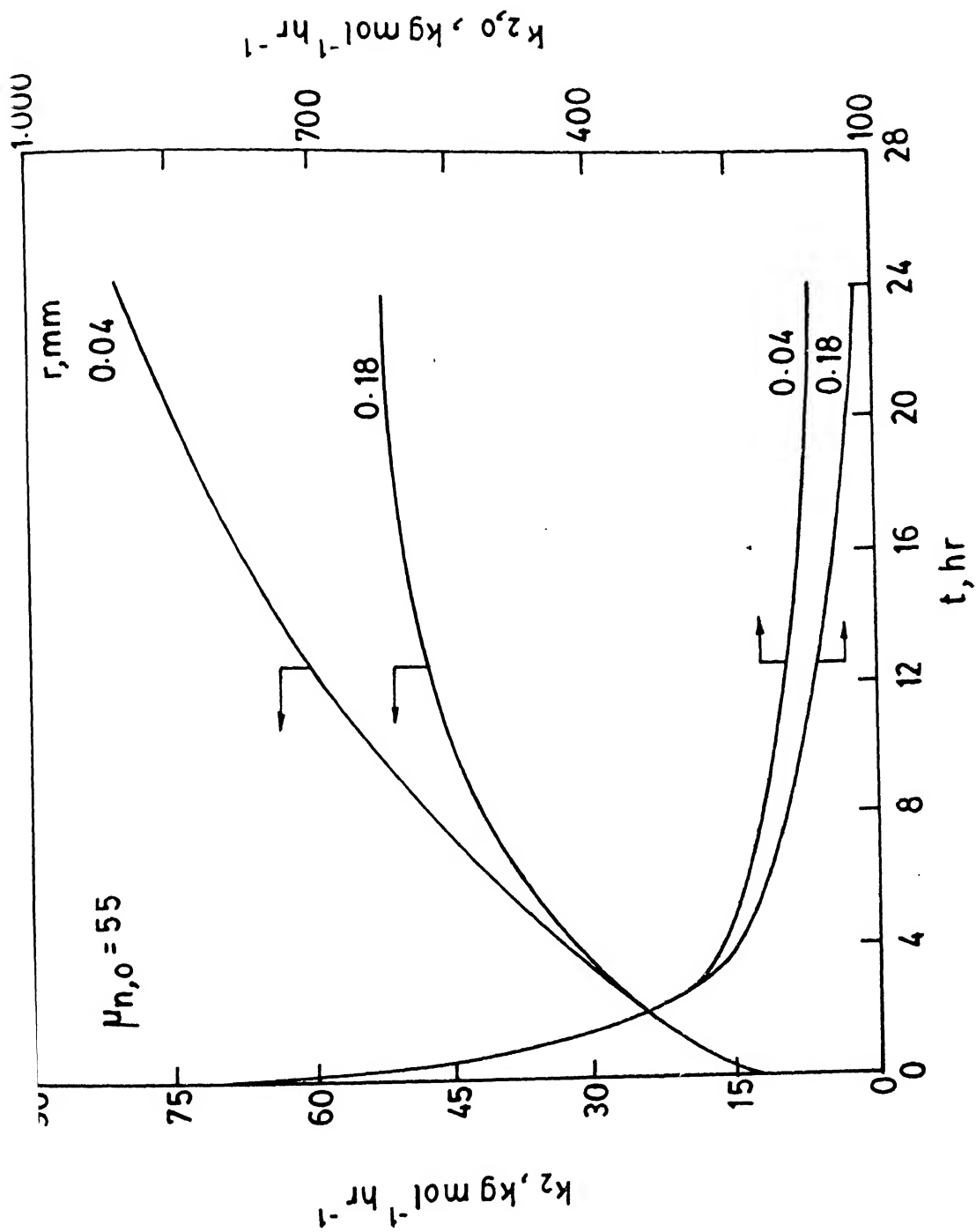


FIG.16
 variation of k_2 and $k_{2,o}$ with time at two radial positions ($r = 0.04, 0.18 \text{ mm}$) for the reference run.

decrease with time at both these locations. The latter is associated with a drop in the value of λ_o (see Eq. 17) which, in turn, is due to the increase in the value of μ_n with time. Fig. 13 shows that μ_n is higher at $r = 0.18$ mm, and so λ_o and $k_{2,o}$ would be expected to be lower at this location than at $r = 0.04$ mm. This is indeed confirmed in Fig. 16. Fig. 17 shows that the ratio, $k_2/k_{2,o}$, increases with time at both locations. This increase indicates that the diffusional resistance decreases as the SSP progresses. This behavior is in sharp contrast to the increase in the diffusional resistance in the polymerization of methyl methacrylate (gel effect). The decreasing diffusional resistance in the SSP of nylon 6 is associated with the decrease in the volume fraction, ϕ_p , of the polymer with time due to generation of C_1 and W . In fact, Figs. 12, 14 and 17 show that as t increases, $[C_1]$ increases at both locations but $[W]$ increases continuously at $r = 0.04$ mm only ($[W]$ decreases with time at $r = 0.18$ mm after an initial rise). This leads to ϕ_p being higher at $r = 0.18$ mm. This complex interplay of several phenomena--generation of C_1 and W , the macro-diffusion of W , decrease of λ_o and $k_{2,o}$ with polymerization, etc., in influencing the rate constant, k_2 , is clearly brought out in these figures. Similar increases in $k_3/k_{3,o}$ and $k'_2/k'_{2,o}$ with time are also observed.

It may be mentioned here that most of the simulation results reported herein have been generated using the closure conditions given in Table IV. Use of $[P_2] = [P_1]$ gives negative monomer concentrations for some extreme choices of parameters which are far removed from the reference values, e.g., for very low ζ_1 and

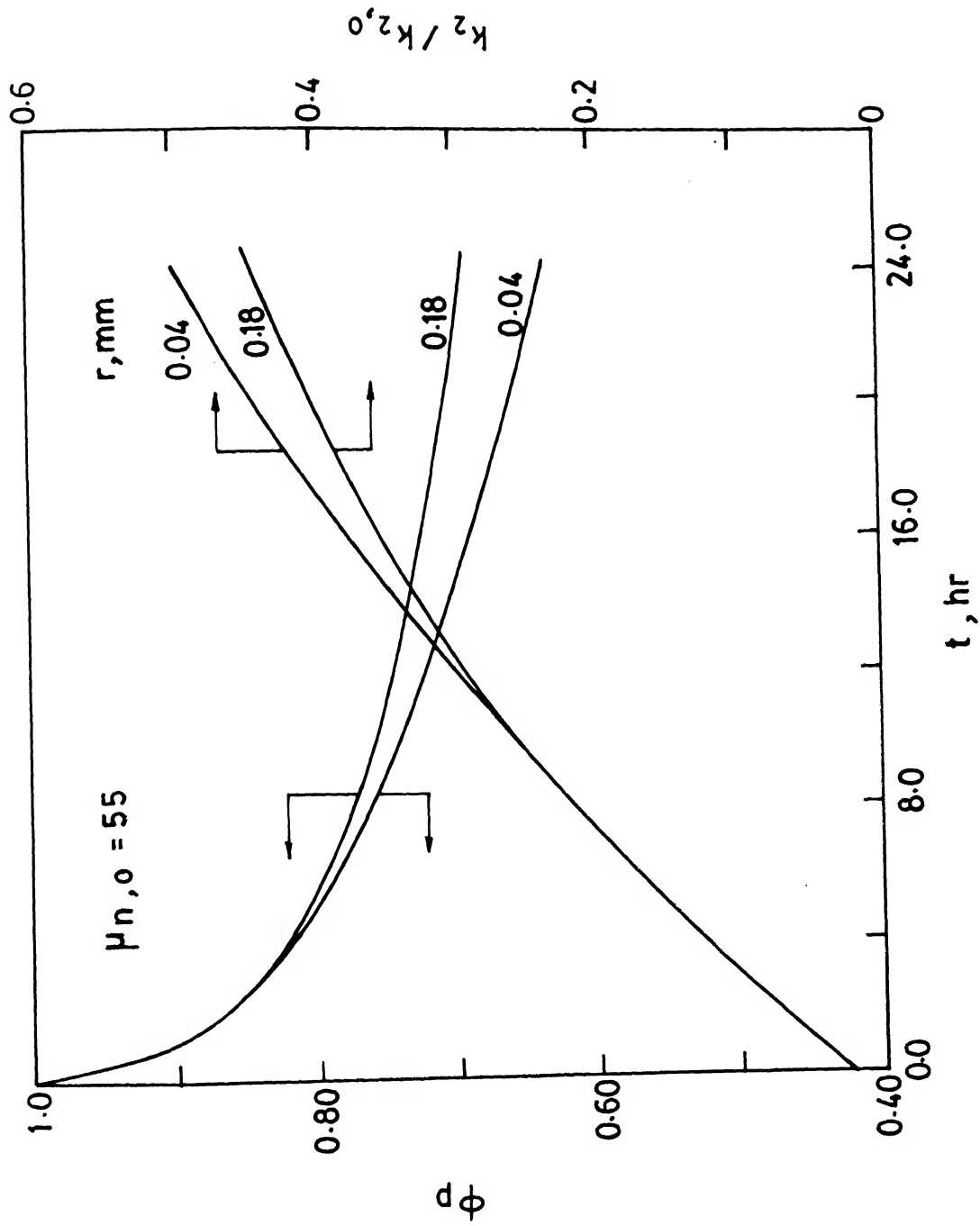


FIG. 17

Variation of ϕ_p and $k_2/k_{2,0}$ with time at two radial positions ($r = 0.04, 0.18$ mm) for the reference run.

very high ζ_3 . Whenever this is encountered, we have used the closure condition, $[P_2] = 0.001 [P_1]$. The moment closure equation in Table IV is used unchanged. Results were regenerated using this new closure equation for all the other parameter values too and were found to be insensitive to this change. We recommend the use of this new closure condition in future studies.

CHAPTER 4

CONCLUSIONS

A new model having a strong molecular basis has been proposed to account for the effects of segmental diffusion on reversible step growth polymerizations. This model is more fundamental and general than earlier models on SSP and can be used to analyze a variety of SSP processes, as for example, nylon 6 polyethylene terephthalate, polybutylene terephthalate, nylon 66, etc. The model is applied to the specific case of the SSP of nylon 6 in this study. A comparison of the present results with our earlier work¹⁰ reveals that agreement of this theory with available experimental data at 190°C is far better.

The optimal values of six parameters, ζ_i and θ_i ; $i = 1, 2, 3$ (or five, with $\zeta_1 = 0$ and $\theta_3 \rightarrow \infty$) obtained in this study can be used at 190°C (alongwith the model equations and the new closure condition developed in this work) to design and optimize industrial nylon 6 SSP reactors in the future. The seventh parameter, D_w , used in this study depends on the crystallinity of the sample, and must be estimated using pilot-plant studies, till such time that a better understanding relating it to the morphology (which could be independently studied) becomes available. It may be added that similar optimal-parameter estimation could be performed for SSP of other polymers (or for nylon 6 at other temperatures), to give a more rational design methodology for those reactors as well. This study quantifies the effects of three important operation parameters, namely, radius of the polymer particle, water concentration in the vapor phase, and the degree of crystallinity of nylon 6 on the spatially-averaged molecular weight history.

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INPUT FOR OPTIMIZATION OF PARAMETERS

I. BOX-COMPLEX ALGORITHM PARAMETERS

$$\alpha = 1.3$$

$$\beta = 0.0001$$

$$\gamma = 5$$

$$\delta = 0.00001$$

$$IC = 1$$

$$IPRINT = 1$$

$$N = 7$$

$$M = 7$$

$$K = 8$$

II. FOR THE FORM $k_{i,o} = \zeta_i \lambda_o$ OF INTRINSIC RATECONSTANTS

1. Initial Guess Values of Parameters

$$\theta_1 = 3.25 \times 10^{-4}$$

$$\theta_2 = 380.0$$

$$\theta_3 = 1475.0$$

$$\zeta_1 = 7.25 \times 10^{-2}$$

$$\zeta_2 = 4150.0$$

$$\zeta_3 = 135.0$$

$$D_w = 4.3 \times 10^{-11} \text{ m}^2/\text{hr}$$

2. Range of Parameters

| | Lower bound | Upper bound |
|-------------------------------|----------------------|----------------------|
| θ_1 | 1.0×10^{-4} | 6.0×10^{-4} |
| θ_2 | 5.0 | 1000.0 |
| θ_3 | 10.0 | 10000.0 |
| ζ_1 | 1.0×10^{-2} | 100.0 |
| ζ_2 | 1.0×10^3 | 20×10^3 |
| ζ_3 | 5.0 | 800.0 |
| $D_w \text{ (m}^2/\text{hr)}$ | 1.0×10^{11} | 5.0×10^{11} |

$$3. \text{ Final Error (\%)} = 3.78$$

III. FOR THE FORM $k_{i,o} = \zeta_i$ OF INTRINSIC RATECONSTANTS

1. Initial Guess Values of Parameters

$$\begin{aligned}\theta_1 &= 4.65 \times 10^{-4} \\ \theta_2 &= 250.0 \\ \theta_3 &= 635.0 \\ \zeta_1 &= 0.0113 \\ \zeta_2 &= 331.0 \\ \zeta_3 &= 13.5 \\ D_w &= 3.4 \times 10^{-11} \text{ m}^2/\text{hr}\end{aligned}$$

2. Range of Parameters

| | Lower bound | Upper bound |
|-------------------------------|-----------------------|-----------------------|
| θ_1 | 1.0×10^{-4} | 6.0×10^{-4} |
| θ_2 | 5.0 | 1000.0 |
| θ_3 | 10.0 | 5000.0 |
| ζ_1 | 0.001 | 10.0 |
| ζ_2 | 10 | 1000.0 |
| ζ_3 | 1.0 | 50.0 |
| $D_w \text{ (m}^2/\text{hr)}$ | 1.0×10^{-11} | 5.0×10^{-11} |

3. Final Error (%) = 5.06

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=====

SIMULATION OF SOLID STATE POLYMERIZATION OF NYLON 6

-[MUKUND R. KULKARNI_9210212]

=====

Using d02ebfe.f routine from NAG library.

=====

FEW ASPECTS :

1. ALL THREE REACTIONS CONSIDERED.
2. MACRO-LEVEL DIFFUSIVITY OF WATER IS A PARAMETER.
3. $[P_2] = 0.001[P_1]$ IS THE CLOSURE CONDITION.
4. USE $icase = 1$ for $k_{i,0} = zeta_{i1} \cdot lam_0$
 $icase = 2$ for $k_{i,0} = zeta_{i1}$
5. No. of grid points $np = 11$.

INPUT FILES :

'input' : Give initial conditions

'input1' : give $icase$ & values of the parameters

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DOUBLE PRECISION NINDEX

.. Parameters ..

PARAMETER (N=66,IW=21*N+23,NP=11)

PARAMETER (N=66,IW=5500,NP=11)

PARAMETER (NOUT=6)

.. Local Arrays ..

DOUBLE PRECISION W(IW), Y(N)

.. External Functions ..

EXTERNAL D02EJW, G

.. External Subroutines ..

EXTERNAL D02EJY, FCN, OUT, PEDERV

EXTERNAL D02EBF

.. Intrinsic Functions ..

INTRINSIC DBLE

DIMENSION STATEMENTS

DIMENSION A0(3),E0(3),AC(3),EC(3),DH(3),DS(3)

COMMON STATEMENTS

COMMON XEND, H, I

COMMON /B1/Y

COMMON /B2/R(NP),FKS2(NP),FKS3(NP),RKS2(NP),RKS3(NP)

COMMON /B3/EK2,EK3

COMMON /B4/ZETA2,ZETA3

COMMON /B5/DW,TEMP,WS,DR

COMMON /B6/RR,PI

COMMON /B9/THETA,AT,BT,TGINFINITY,P1,ALPHAL,ALPHAG

COMMON /B11/RADIUS

COMMON /B13/TG(NP),VF(NP),DD0(NP)

COMMON /B14/FX1(NP),FX2(NP),FX3(NP),FX4(NP),FX5(NP),FX6(NP)

COMMON /B15/AVEC1,AVEP1,AVELAM0,AVELAM1,AVELAM2,AVEM

COMMON /B16/AVEVF,AVETG

COMMON /B18/VMS,VPS,GAMMA,VN0,ZETA,VFM

COMMON /B19/NINDEX,WM,WSOL

COMMON /B20/THETA1,THETA2,THETA3

```

COMMON /B21/FKS1(NP),RKS1(NP)
COMMON /B22/EK1,ZETA1
COMMON /B23/DD02(np),DD03(np)
COMMON /B24/VS0,VFS,VSS
COMMON /B25/ZETA13,ZETA23
  common /const/const
  common /case/ibase

```

OPEN STATEMENTS

```

OPEN(UNIT = 12, FILE = 'Ta1etal')
OPEN(UNIT = 14, FILE = 'input')
OPEN(UNIT = 15, FILE = 'input1')
OPEN(UNIT = 20, FILE = 'output')
OPEN(UNIT = 21, FILE = 'ssp.o1')
OPEN(UNIT = 22, FILE = 'ssp.o2')
OPEN(UNIT = 23, FILE = 'pdi')
OPEN(UNIT = 24, FILE = 'rateconst')

```

.. Executable Statements ..

.... READING INPUT DATA

```

DO IJ = 1,3
  READ(12,*)A0(IJ),E0(IJ),AC(IJ),EC(IJ),DH(IJ),DS(IJ)
END DO
  READ(14,*)Y10,Y20,Y30,Y40,Y50,Y60

```

.... CONSTANTS

RR : GAS CONSTANT J/K/gmol

Nav : AVA. NO.

PI : 3.1417

RR = 8.313D0

.... BOUNDARY CONDITION

WS = 0.00001d0

.... INITIAL CONDITIONS

```

DO JJ = 1,Np
Y(JJ) = Y10
Y(Np+JJ) = Y20
Y(2*NP+JJ) = Y30
Y(3*NP+JJ) = Y40
Y(4*NP+JJ) = Y50
Y(5*NP+JJ) = Y60
IF(JJ.EQ.NP)THEN
Y(5*NP+JJ) = WS
ENDIF
END DO

```

TEMP = 463.16D0

TGINFINITY = 350.0D0

P1 = 0.03D0

ALPHAL = 1.0D-3

ALPHAG = 4.5D-4

RADIUS = 0.02D0

NINDEX=2.0d0

```

const = 0.001d0
-----READING PARAMETER VALUES-----
read(15,*)icase
read(15,*)THETA1,THETA2,THETA3,ZETA1,ZETA2,ZETA3,DW,TOL
write(21,*)'THETA1,THETA2,THETA3,ZETA1,ZETA2,ZETA3,DW,TOL'
write(21,*)THETA1,THETA2,THETA3,ZETA1,ZETA2,ZETA3,DW,TOL
WRITE(20,331)
1  format(1x,'TIME',6x,'AVEC1',6x,'AVEP1',4x,'AVELAM0',3x,
+  'AVELAM1',2x,'AVELAM2'
+  4x,'AVEW',
+  6x,'AVEMUN')
-----FREE VOLUME PARAMETERS-----
VMS = 0.864263D0
VPS = 0.81232D0
VSS = 0.969D0
ZETA = 0.709296d0
GAMMA = 1.00D0
VM0 = 1.13247D0
VFM = 0.31033D0
VS0 = 1.1429D0
VFS = 0.1795d0

WM = 113.16D0
WSOL = 18.0D0
ZETA13 = VMS/VPS*(2.0/3.0)
ZETA23 = (VSS/VPS)*(WS/WM)*(2.0/3.0)
ZETA13 = 0.709296D0
ZETA23 = 0.1265D0

X = 0.0D0
-----EQUILIBRIUM CONSTANTS-----
EK1 = 2.351698D-03
EK2 = DEXP((DS(2)-DH(2)/TEMP)/RR)
EK3 = DEXP((DS(3)-DH(3)/TEMP)/RR)
EK2 = 1033.2474d0
EK3 = 2.456d0
-----
DR = 0.002d0
DO JK = 1,NP
R(JK) = DR*(JK - 1)
END DO
-----
WRITE (NOUT,*) '--- NYLON 6 SSP SIMULATION ---'
XEND =24.0D0
MPED = 0
IR = 2

WRITE (NOUT,99999) ' Calculation with TOL =', TOL
I = 23
H = (XEND-X)/DBLE(I+1)
IFAIL = 0
CALL D02EBF(X,XEND,N,Y,TOL,IR,FCN,MPED,D02EJY,OUT,W,IW,IFAIL)
IF (TOL.LT.0.0D0) WRITE (NOUT,*) ' Range too short for TOL'

```

55 STOP

```
*
99999 FORMAT (1X,A,D8.1)
99998 FORMAT (1X,A,F7.3)
99997 FORMAT (1X,A,6(((11E16.5)/))
99996 FORMAT (F4.2/6(((11E16.5)/))
END
```

```
*
-----
SUBROUTINE FCN(T,Y,F)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION NINDEX
PARAMETER (N=66,Np=11)
DIMENSION F(N), Y(N)
COMMON /B2/R(Np),FKS2(Np),FKS3(Np),RKS2(Np),RKS3(Np)
COMMON /B5/DW,TEMP,WS,DR
COMMON /B21/FKS1(NP),RKS1(NP)
common /const/const
c .. Executable Statements ..
```

DR = 0.002D0

DO 25 JJ = 1,Np

```
IJ1 = JJ
IJ2 = 1*Np + JJ
IJ3 = 2*Np + JJ
IJ4 = 3*Np + JJ
IJ5 = 4*Np + JJ
IJ6 = 5*Np + JJ
```

CALL RATECONST(JJ,IJ3,IJ4,IJ6,Y)

```
ZZZ = +FKS1(JJ)*Y(IJ1)*Y(IJ6) - RKS1(JJ)*Y(IJ2)
XL3 = Y(IJ5)* ( 2.0D0*Y(IJ5)*Y(IJ3) - Y(IJ4)*Y(IJ4) )
* / (Y(IJ4)*Y(IJ3))
F(IJ1) = -FKS3(JJ)*Y(IJ1)*Y(IJ3)
* +RKS3(JJ)*(Y(IJ3) - Y(IJ2)) - ZZZ
F(IJ2) = -2.0D0*FKS2(JJ)*Y(IJ2)*Y(IJ3)
* +2.0D0*RKS2(JJ)*Y(IJ6)*(Y(IJ3) - Y(IJ2))
* -FKS3(JJ)*Y(IJ2)*Y(IJ1)
* +RKS3(JJ)*const*Y(IJ2) + ZZZ
F(IJ3) = -FKS2(JJ)*Y(IJ3)*Y(IJ3)
* +RKS2(JJ)*Y(IJ6)*(Y(IJ4) - Y(IJ3))
* +ZZZ
F(IJ4) = FKS3(JJ)*Y(IJ1)*Y(IJ3) -
* RKS3(JJ)*(Y(IJ3) - Y(IJ2))
* +ZZZ
F(IJ5) = 2.0D0*FKS2(JJ)*Y(IJ4)*Y(IJ4)
* +1.0D0/3.0D0*RKS2(JJ)*Y(IJ6)*(Y(IJ4) - XL3)
* +FKS3(JJ)*Y(IJ1)*Y(IJ3) + 2.0D0*Y(IJ4))
* +RKS3(JJ)*(Y(IJ3) - 2.0D0*Y(IJ4) + Y(IJ2))
* +ZZZ
IF(JJ.EQ.1) THEN
YP = Y(57)
```



```

F(IJ6) = -F(IJ3)
*      +Dw*((Y(IJ6+1)-2.0D0*Y(IJ6)+YP)/(DR*DR))
ELSE
YP = Y(IJ6-1)
F(IJ6) = -F(IJ3)
*      +Dw*((Y(IJ6+1)-2.0D0*Y(IJ6)+YP)/(DR*DR))
*      +Dw*(Y(IJ6+1) - YP)/R(JJ)/DR
endif
F(66) = 0.0d0
25  CONTINUE

```

```

RETURN
END

```

```

=====
SUBROUTINE OUT(X,Y)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION NINDEX
* .. Parameters ..
PARAMETER      (N=66,NP=11)
PARAMETER      (NOUT=6)
* .. Array Arguments ..
DOUBLE PRECISION Y(N)
DOUBLE PRECISION RRR(NP)

COMMON  XEND, H, I
COMMON /B2/R(NP),FKS2(NP),FKS3(NP),RKS2(NP),RKS3(NP)
COMMON /B11/RADIUS
COMMON /B13/TG(NP),VF(NP),DD0(NP),AMUN(NP)
COMMON /B14/FX1(NP),FX2(NP),FX3(NP),FX4(NP),FX5(NP),FX6(NP)
COMMON /B15/AVEC1,AVEP1,AVELAM0,AVELAM1,AVELAM2,AVEN
COMMON /B16/AVEVF,AVETG
COMMON /B17/FX7(NP),FX8(NP),FX9(NP)
COMMON /B21/FKS1(NP),RKS1(NP)
COMMON /B4/ZETA2,ZETA3
COMMON /B20/THETA1,THETA2,THETA3

WRITE(22,*)'TIME=',X
DO JJ = 1,NP
IJ1    = JJ
IJ2    = 1*Np + JJ
IJ3    = 2*Np + JJ
IJ4    = 3*Np + JJ
IJ5    = 4*Np + JJ
IJ6    = 5*Np + JJ
  RRR(JJ) = R(JJ)*R(JJ)
  CALL RATECONST(JJ,IJ3,IJ4,IJ6,Y)
  FX1(JJ) = RRR(JJ)*Y(IJ1)
  FX2(JJ) = RRR(JJ)*Y(IJ2)
  FX3(JJ) = RRR(JJ)*Y(IJ3)
  FX4(JJ) = RRR(JJ)*Y(IJ4)
  FX5(JJ) = RRR(JJ)*Y(IJ5)
  FX6(JJ) = RRR(JJ)*Y(IJ6)
  FX7(JJ) = AMUN(JJ)

```

```

      FX8(JJ) = RRR(JJ)*VF(JJ)
      FX9(JJ) = RRR(JJ)*TG(JJ)
      WRITE(20,19)X,Y(IJ1),Y(IJ2),Y(IJ3),Y(IJ4)
+      ,Y(IJ5),Y(IJ6),AMUN(JJ)
19      FORMAT(8(E9.3,1X))
      WRITE(22,*)R(JJ), '      ', amun(jj)

```

END DO

CALL AVERAGE(Y,AVEHUN)

avepdi = avelam2*avelamo/(avelam1**2)

WRITE(6,*)'TIME= ',X, ' AVEHUN: ', AVEHUN, ' PDI: ', avepdi

WRITE(20,29)X,AVEC1,AVEP1,AVELAMO,AVELAM1,AVELAM2,AVEW,AVEMUN

29 FORMAT(8(E9.3,1X))

WRITE(21,22)X,AVEMUN

WRITE(23,22)X,avepdi

22 FORMAT(4X,F10.3,4X,F10.4)

X = XEND - DBLE(I)*H

I = I - 1

RETURN

*
99999 FORMAT (F10.4/6((6F16.8/5F16.8)/))
END

=====

SUBROUTINE RATECONST(JJ,IJ3,IJ4,IJ6,Y)

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DOUBLE PRECISION NINDEX

PARAMETER (N=66,Np=11)

DIMENSION Y(N),FKS20(NP),FKS30(NP),RKS20(NP)

COMMON /B2/R(Np),FKS2(Np),FKS3(Np),RKS2(Np),RKS3(Np)

COMMON /B3/EK2,EK3

COMMON /B4/ZETA2,ZETA3

COMMON /B9/THETA,AT,BT,TGINFINITY,P1,ALPHAL,ALPHAG

COMMON /B13/TG(NP),VF(NP),DD0(NP)

COMMON /B20/THETA1,THETA2,THETA3

COMMON /B21/FKS1(NP),RKS1(NP)

COMMON /B22/EK1,ZETA1

COMMON /B23/DD02(np),DD03(np)

COMMON /B25/ZETA13,ZETA23

common /case/ibase

CALL DIFFUSIVITY1(Y,JJ,ij3,ij4,ij6)

XYZ = THETA/DD0(JJ)

XYZ1 = THETA1/DD0(JJ)

XYZ2 = THETA2/DD02(JJ)

XYZ3 = THETA3/DD03(JJ)

FKS1(JJ) = ZETA1

FKS20(JJ) = ZETA2

FKS30(JJ) = ZETA3

```

if(licase.ne.1)go to 222
FKS1(JJ) = ZETA1*Y(IJ3)
FKS20(JJ) = ZETA2*Y(IJ3)
FKS30(JJ) = ZETA3*Y(IJ3)

```

```

222      RKS1(JJ) = FKS1(JJ)/EK1
      RKS20(JJ) = FKS20(JJ)/EK2
      RKS3(JJ) = FKS30(JJ)/EK3

```

```

C-----diffusion affected rate constants-----
FKS2(JJ) = 1.0D0/((1.0D0/(FKS20(JJ)) + XYZ1*Y(IJ3))
RKS2(JJ) = 1.0D0/((1.0D0/(RKS20(JJ)) + XYZ2*Y(IJ3))
FKS3(JJ) = 1.0D0/((1.0D0/(FKS30(JJ)) + XYZ3*Y(IJ3))

```

```

RETURN
END

```

```

=====
SUBROUTINE DIFFUSIVITY1(Y,jj,ij3,ij4,ij6)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION NINDEX
PARAMETER(N=66, NP=11)

```

```

DIMENSION Y(N),DD(NP),DD1(NP),DD2(NP)
COMMON /B5/DW,TEMP,WS,DR
COMMON /B9/THETA,AT,BT,TGINFINITY,P1,ALPHAL,ALPHAG
COMMON /B11/RADIUS
COMMON /B13/TG(NP),VF(NP),DD0(NP),AMUN(NP)
COMMON /B18/VMS,VPS,GAMMA,VM0,ZETA,VFM
COMMON /B19/NINDEX,WM,WSOL
COMMON /B20/THETA1
COMMON /B23/DD02(np),DD03(np)
COMMON /B24/VS0,VFS,VSS
COMMON /B25/ZETA13,ZETA23

```

```

AMUN(JJ) = Y(IJ4)/Y(IJ3)
TG(JJ) = 1.0D0/((1.0D0/TGINFINITY + P1/AMUN(JJ))
VF(JJ) = 0.025 + (ALPHAL - ALPHAG)*(TEMP - TG(JJ))
DD0(JJ) = DEXP(2.303*VF(JJ)/(AT + BT*VF(JJ)))

```

```

VP0 = VPS*(1.0+VF(JJ))
TV = (Y(JJ)*WM*VM0 + Y(IJ3)*AMUN(JJ)*WM*VP0 + Y(IJ6)*WSOL*VS0)
PHIM = Y(JJ)*WM*VM0/TV
PHIS = Y(IJ6)*WSOL*VS0/TV
PHIP = Y(IJ3)*AMUN(JJ)*WM*VP0/TV
DM = 1.0/VM0
DP = 1.0/VP0
DS = 1.0/VS0
VFP = VF(JJ)
XNUM = GAMMA*(DM*PHIM*VMS/ZETA13 + DS*PHIS*VSS/ZETA23 +
* DP*PHIP*VPS)
XDEN = DM*PHIM*VMS*VFM + DS*PHIS*VSS*VFS +DP*PHIP*VPS*VFP
CHI = XNUM/XDEN
CHIO = GAMMA/VFP

```

```

DD(JJ) = DEXP(-CHI + CHI0)
DD02(JJ) = DEXP(ZETA23*(-CHI+CHI0))
DD03(JJ) = DEXP(ZETA13*(-CHI+CHI0))
DD0(JJ) = (AMUN(JJ)**(-NINDEX)) * DD(JJ)
RETURN
END

```

```

=====
SUBROUTINE AVERAGE(Y,AVEMUN)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DOUBLE PRECISION NINDEX
PARAMETER (N=66,NP=11)
DIMENSION Y(N)
COMMON /B11/RADIUS
COMMON /B13/TG(NP),VF(NP),DD0(NP),AMUN(NP)
COMMON /B14/FX1(NP),FX2(NP),FX3(NP),FX4(NP),FX5(NP),FX6(NP)
COMMON /B17/FX7(NP),FX8(NP),FX9(NP)
COMMON /B15/AVEC1,AVEP1,AVELAMO,AVELAM1,AVELAM2,AVEN
COMMON /B16/AVEVF,AVETG

```

```

DENO = (0.02**3.0)/3.0D0
CALL INTEGRATION(FX1,0.0,0.02,11,AINT1)
CALL INTEGRATION(FX2,0.0,0.02,11,AINT2)
CALL INTEGRATION(FX3,0.0,0.02,11,AINT3)
CALL INTEGRATION(FX4,0.0,0.02,11,AINT4)
CALL INTEGRATION(FX5,0.0,0.02,11,AINT5)
CALL INTEGRATION(FX6,0.0,0.02,11,AINT6)
CALL INTEGRATION(FX7,0.0,0.02,11,AINT7)
CALL INTEGRATION(FX8,0.0,0.02,11,AINT8)
CALL INTEGRATION(FX9,0.0,0.02,11,AINT9)
AVEC1 = AINT1/DENO
AVEP1 = AINT2/DENO
AVELAMO = AINT3/DENO
AVELAM1 = AINT4/DENO
AVELAM2 = AINT5/DENO
AVEN = AINT6/DENO
AVEVF = AINT8/DENO
AVETG = AINT9/DENO
AVEMUN = AINT4/AINT3

```

```

RETURN
END

```

```

=====
SUBROUTINE INTEGRATION(FX,XI,XL,NPOINTS,XINT)

```

THIS SUBROUTINE INTEGRATES FUNCTION 'FX' OVER THE RANGE 'XI' TO 'XL'
 'NPOINTS-1' NO. OF INTERVALS. NPOINTS SHOULD BE ODD!
 'XINT' IS THE VALUE OF INTEGRAL USING SIMPSONS' RULE.

```

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
PARAMETER (NP=11)
DIMENSION FX(NP)

```

```

N1 = NPOINTS - 1
N2 = NPOINTS - 2

```

DX = 0.002

SUM1 = 0.0

DO JJ = 3,N2,2

SUM1 = SUM1 + FX(JJ)

END DO

SUM2 = 0.0

DO JJ = 2,N1,2

SUM2 = SUM2 + FX(JJ)

END DO

SUM = FX(1) + FX(NPOINTS) + 2.0D0*SUM1 + 4.0D0*SUM2

XINT = DX*SUM/3.0

RETURN

END

=====

-----OPTIMIZATION USING BOX-COMPLEX ALGORITHM-----

MAIN LINE PROGRAM FOR COMPLEX ALGORITHM OF BOX

IMPLICIT DOUBLE PRECISION(A-H,O-Z)

PARAMETER(N=7,M=7,K=8)

Double precision X(K,M),F(K),G(M),H(M),XC(N)

INTEGER GAMMA

COMMON /B27/YINI(5,6)

COMMON /B28/EXMUN(5,13),NSET

COMMON /B30/ERROR,TOL

COMMON /B31/IT,ITMAX

common /bb/ichoice

common /const/const

OPEN STATEMENTS

OPEN(UNIT = 14, FILE = 'input')

OPEN(UNIT = 33, FILE = 'exdata')

OPEN(UNIT = 21, FILE = 'ssp.01')

OPEN(UNIT=55,FILE='guess')

OPEN(UNIT=59,FILE='opt')

OPEN(UNIT=60,FILE='out007')

NO=59

INPUT FILES :

'input' : Initial Concentrations for All the Sets

'exdata' : Experimental Data (5 Sets)

'opt' : output

const = 0.001d0

DATA IC,IPRINT/1,1/

DATA ALPHA,BETA,GAMMA,DELTA/1.3,0.0001,5,0.000001/

read(55,*)ichoice

if(ichoice.eq.0)go to 122

do ijk = 1,K

READ(55,*)(X(ijk,JJ),JJ=1,N)

end do

go to 558

READ(55,*)(X(1,JJ),JJ=1,N)

READ(14,*)nset,TOL,ITMAX

do 77 iset =1,nset

read(14,*)(YINI(iset,jk),jk=1,6)

read(33,*)(EXMUN(iset,ij),ij=1,13)

continue

WRITE(NO,010)

FORMAT(/,18X,'COMPLEX PROCEDURE OF BOX')

WRITE(NO,018)

FORMAT(/,2X,'PARAMETERS')

WRITE(NO,011)N,M,K,ITMAX,IC,ALPHA,BETA,GAMMA,DELTA

FORMAT(/,2X,'N=',14,3X,'M=',14,3X,'K=',14,2X,'ITMAX=',

```

1 16,2X,'IC=',I4,/,2X,'ALPHA=',F8.4,5X,'BETA=',F10.5,3X,
2 'GAMMA=',I4,3X,'DELTA=',F10.5)

```

```

3 CALL CONSX(N,M,K,ALPHA,BETA,GAMMA,DELTA,X,F,IEV2,
NO,G,H,XC,IPRINT)
IF(IT-ITMAX)20,20,30
WRITE(NO,014)F(IEV2)
FORMAT(///,2X,'FINAL VALUES OF THE FUNCTION =',E20.8)
WRITE(NO,015)
FORMAT(///,2X,'FINAL X VALUES')
DO 300 J=1,N
WRITE(NO,*) J,X(IEV2,J)
6 FORMAT(/,2X,'J=',I4,5X,'X=',E20.8)
CONTINUE
GO TO 999
WRITE(NO,017) ITMAX
FORMAT(///,2X,'THE NUMBER OF ITERATIONS HAS EXCEEDED',I4
2 'PROGRAM TERMINATED')
STOP
END

```

```

*****
SUBROUTINE CHECK(N,M,K,X,G,H,I,KODE,XC,DELTA,K1)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
Double precision X(K,M),G(M),H(M),XC(N)
INTEGER GAMMA

```

```

    KT=0
CALL CONST(N,M,K,X,G,H,I)

```

CHECK AGAINST EXPLICIT CONSTRAINTS

```

DO 50 J=1,N
IF(X(I,J)-G(J))20,20,30
X(I,J)=G(J)+DELTA
GO TO 50
IF(H(J)-X(I,J))40,40,50
X(I,J)=H(J)-DELTA
CONTINUE
IF(KODE)110,110,60
NN=N+1
DO 100 J =NN,M
CALL CONST(N,M,K,X,G,H,I)
IF(X(I,J)-G(J))80,70,70
IF(H(J)-X(I,J))80,100,100
IEV1=I
KT=1
CALL CENTR(N,M,K,IEV1,I,XC,X,K1)
DO 90 JJ =1,N
X(I,JJ)=(X(I,JJ)+XC(JJ))/2.0
CONTINUE
CONTINUE
IF(KT)110,110,10
RETURN
END

```

SUBROUTINE CENTR(N,M,K,IEV1,I,XC,X,K1)

IMPLICIT DOUBLE PRECISION(A-H,O-Z)

Double precision X(K,M),XC(N)

DO 20 J = 1, N

XC(J)=0.0

DO 10 IL =1,K1

XC(J)=XC(J)+X(IL,J)

RK=K1

XC(J)=(XC(J)-X(IEV1,J))/(RK-1.0)

RETURN

END

SUBROUTINE CONSX(N,M,K,ALPHA,BETA,GAMMA,DELTA,X,

F,IEV2,NO,G,H,XC,IPRINT)

COORDINATES SPECIAL PURPOSE SUBROUTINES

ARGUMENT LIST

IT=ITERATION INDEX

IEV1=INDEX OF POINT WITH MIN. FUNCTION VALUE

IEV2=INDEX OF POINT WITH MAX. FUNCTION VALUE

I=POINT INDEX

KODE= CONTROL KEY USED TO DETERMINE IF IMPLICIT CONSTRAINTS
ARE PROVIDED

K1=DO LOOP LIMIT

ALL OTHERS ARE PREVIOUSLY DEFINED IN MAIN LINE

IMPLICIT DOUBLE PRECISION(A-H,O-Z)

INTEGER GAMMA

DIMENSION X(K,M),F(K),G(M),H(M),XC(N)

COMMON /B31/IT,ITMAX

common /bb/ichoice

IT=1

if(ichoice.eq.1)go to 333

DO 40 II =2,K

DO 30 J=1,N

X(II,J)=0.0

CONTINUE

CONTINUE

CALCULATE COMPLEX POINTS CHECK AGAINST CONSTRAINTS

write(6,*)'CALCULATING COMPLEX POINTS CHECK AGAINST CONSTRAINTS

write(60,*)'CALCULATING COMPLEX POINTS '

DO 65 II=2,K

DO 50 J=1,N

I=II

CALL CONST(N,M,K,X,G,H,I)

call G05CCF

RANDOM = G05CAF(XX)

X(II,J)=G(J)+RANDOM*(H(J)-G(J))

CONTINUE


```

K1=I1
CALL CHECK(N,M,K,X,G,H,I,KODE,XC,DELTA,K1)
IF(I1-2)51,51,55
51 IF(IPRINT)52,65,52
52 WRITE(NO,018)
WRITE(60,018)
018 FORMAT(/,2X,'COORDINATES OF INITIAL COMPLEX')
IO=1
WRITE(NO,019)(IO,J,X(IO,J),J=1,N)
WRITE(60,019)(IO,J,X(IO,J),J=1,N)
019 FORMAT(/,3(2X,2HX(,I2,1H,,I2,4H) = ,E13.6))
55 IF(IPRINT)56,65,56
56 WRITE(NO,019)(I1,J,X(I1,J),J=1,N)
65 CONTINUE
333 K1=K
DO 70 I =1,K
CALL FUNC(N,M,K,X,F,I)
70 CONTINUE
KOUNT=1
IA=0

```

```

C FIND POINT OF LOWEST FUNCTION VALUE
write(6,*)'FIND POINT OF LOWEST FUNCTION VALUE'
write(60,*)'FIND POINT OF LOWEST FUNCTION VALUE'

```

```

IF(IPRINT)72,80,72
72 WRITE(NO,021)
WRITE(60,021)
WRITE(6,021)
021 FORMAT(/,2X,2HVALUES OF THE FUNCTION )
WRITE(NO,022)(J,F(J),J=1,K)
WRITE(60,022)(J,F(J),J=1,K)
WRITE(6,022)(J,F(J),J=1,K)
022 FORMAT(/,3(2X,2HF(,I2,4H) = ,1PE13.6))
80 IEV1=1
DO 100 ICM =2,K
IF(F(IEV1)-F(ICM))100,100,90
90 IEV1=ICM
100 CONTINUE

```

```

C FIND POINT WITH HIGHEST FUNCTION VALUE
write(6,*)'FIND POINT WITH HIGHEST FUNCTION VALUE'
write(60,*)'FIND POINT WITH HIGHEST FUNCTION VALUE'

```

```

IEV2=1
DO 120 ICM =2,K
IF(F(IEV2)-F(ICM))110,110,120
110 IEV2=ICM
120 CONTINUE

```

```

C CHECK CONVERGENCE CRITERIA
write(6,*)'CHECK CONVERGENCE CRITERIA'
write(60,*)'CHECK CONVERGENCE CRITERIA'

```

```

130 IF(F(IEV2)-(F(IEV1)+BETA))140,130,130
    KOUNT=1
    GO TO 150
140 KOUNT=KOUNT+1
    IF(KOUNT-GAMMA)150,240,240

C    REPLACE POINT WITH LOWEST FUNCTION VALUE
    write(6,*)'REPLACE POINT WITH LOWEST FUNCTION VALUE'
    write(60,*)'REPLACE POINT WITH LOWEST FUNCTION VALUE'

150 CALL CENTR(N,M,K,IEV1,I,XC,X,K1)
    DO 160 JJ =1,N
160 X(IEV1,JJ)=(1.0+ALPHA)*(XC(JJ))-ALPHA*(X(IEV1,JJ))
161 I=IEV1
    CALL CHECK(N,M,K,X,G,H,I,KODE,XC,DELTA,K1)
    CALL FUNC(N,M,K,X,F,I)
    write(6,*)'F VALUE:', F(IEV1)

C    REPLACE NEW POINT IF IT REPEATS AS LOWEST FUNCTION VALUE
    write(6,*)'REPLACE NEW POINT REPEAT LOWEST FUNC VALUE'
    write(60,*)'REPLACE NEW POINT REPEAT LOWEST FUNC VALUE'

170 IEV2=1
    DO 190 ICM =2,K
    IF(F(IEV2)-F(ICM))190,190,180
180 IEV2=ICM
190 CONTINUE
    IF(IEV2-IEV1)220,200,220
200 DO 2100 JJ =1,N
    X(IEV1,JJ)=(X(IEV1,JJ)+XC(JJ))/2.0
2100 CONTINUE
    WRITE(60,*)'CORDINATES OF ALL THE POINTS & F VALUES'
    do 9901 mr = 1,K
    write(60,*)(X(mr,jc),jc=1,N),'F(',mr,')',F(mr)
9901 continue

162 I=IEV1
    CALL CHECK(N,M,K,X,G,H,I,KODE,XC,DELTA,K1)
    CALL FUNC(N,M,K,X,F,I)
    write(6,*)'F VALUE:', F(IEV1)
    GO TO 170
220 CONTINUE
    IF(IPRINT)230,228,230
230 WRITE(NO,023)IT
    WRITE(60,023)IT
023 FORMAT(//,2X,17HITERATION NUMBER ,15)
    write(*,*)'ItNo=',IT
    WRITE(NO,024)
    WRITE(60,024)
024 FORMAT(//,2X,'CORDINATES OF CORRECTED POINT')
    WRITE(NO,019)(IEV1,JC,X(IEV1,JC),JC=1,N)
    WRITE(60,019)(IEV1,JC,X(IEV1,JC),JC=1,N)
    WRITE(60,*)'CORDINATES OF ALL THE POINTS & F VALUES'
    do 901 mr = 1,K

```

```

901  write(60,*)(X(mr,jc),jc=1,N),'F(',mr,')',F(mr)
      continue
      WRITE(NO,021)
      WRITE(NO,022)(I,F(I),I=1,K)
      WRITE(NO,025)
      WRITE(60,021)
      WRITE(60,025)
025  FORMAT(/,2X,'COORDINATES OF THE CENTROID')
      WRITE(NO,026)(JC,XC(JC),JC=1,N)
026  FORMAT(/,3(2X,2HX(I,12,6H,C) = ,1PE14.6,4X))
228  IT=IT+1
      IF(IT-ITMAX)80,80,240
240  RETURN
      END

```

```

C *****
SUBROUTINE CONST(N,M,K,X,G,H,I)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
Double precision G(M),H(M),X(K,M)
G(1)= 0.1d0
H(1)= 500.0
G(2)= 1.0
H(2)= 5000.0
G(3)= 1.0
H(3)= 15000.0
G(4)= 0.1
H(4)= 10000.0
G(5)= 1.0
H(5)= 45000.0
G(6)= 1.0
H(6)= 20000.0
G(7)= 1.0
H(7)= 50.0
RETURN
END

```

```

C*****
SUBROUTINE FUNC(N,M,K,X,F,I)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DIMENSION X(K,M),F(K)
COMMON /B4/ZETA1,ZETA2,ZETA3
COMMON /B20/THETA1,THETA2,THETA3
COMMON /B5/DW,TEMP,WS,DR
COMMON /B30/ERROR

THETA1 = X(I,1)*1.0d-4
THETA2 = X(I,2)
THETA3 = X(I,3)
ZETA1 = X(I,4)*1.0d-2
ZETA2 = X(I,5)
ZETA3 = X(I,6)
DW = X(I,7)*1.0d-7

CALL SIMULATION

```

```

F(1) = ERROR
write(*,*)'F VALUE=', ERROR
write(60,*)'F VALUE=', ERROR

```

```

RETURN
END

```

```

C*****
C      SUBROUTINE SIMULATION
C      c using d02ebfe.f routine
C=====
C      c considering first reaction
C      c fk2,rk2,fk3 are diffusion limited.
C      c solvent has been considered.
C=====
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION NINDEX
C      * .. Parameters ..
C      PARAMETER      (N=66,IW=5500,NP=11)
C      PARAMETER      (NOUT=6)
C      * .. Local Arrays ..
C      DOUBLE PRECISION W(IW), Y(N)
C      * .. External Functions ..
C      EXTERNAL      D02EJW, G
C      * .. External Subroutines ..
C      EXTERNAL      D02EBF, D02EJY, FCN, OUT, PEDERV,D02CBF
C      * .. Intrinsic Functions ..
C      INTRINSIC      DBLE
C
C      DIMENSION STATEMENTS
C      DIMENSION A0(3),E0(3),AC(3),EC(3),DH(3),DS(3)
C
C      COMMON STATEMENTS
C      COMMON      XEND, H, I
C      COMMON /B2/R(NP),FKS2(NP),FKS3(NP),RKS2(NP),RKS3(NP)
C      COMMON /B3/EK2,EK3
C      COMMON /B4/ZETA1,ZETA2,ZETA3
C      COMMON /B20/THETA1,THETA2,THETA3
C      COMMON /B5/DW,TEMP,WS,DR
C      COMMON /B8/RR,PI
C      COMMON /B9/THETA,AT,BT,TGINFINITY,P1,ALPHAL,ALPHAG
C      COMMON /B11/RADIUS
C      COMMON /B13/TG(NP),VF(NP),DD0(NP)
C      COMMON /B14/FX1(NP),FX2(NP),FX3(NP),FX4(NP),FX5(NP),FX6(NP)
C      COMMON /B15/AVEC1,AVEP1,AVELAMO,AVELAM1,AVELAM2,AVEW
C      COMMON /B16/AVEVF,AVETG
C      COMMON /B18/VMS,VPS,GAMMA,VM0,ZETA,VFM
C      COMMON /B19/NINDEX,WM,WSOL
C      COMMON /B21/FKS1(NP),RKS1(NP)
C      COMMON /B22/EK1
C      COMMON /B23/DD02(np),DD03(np)
C      COMMON /B24/V80,VF8,V88
C      COMMON /B25/ZETA13,ZETA23
C      COMMON /B27/YINI(5,6)

```

```
COMMON /B28/EXMUN(5,13),NSET,ISET
COMMON /B29/ERRORINI,ICOUNT
COMMON /B30/ERROR,TOL
COMMON /B31/IT,ITMAX
```

```
ERROR = 0.0D0
DO 777 ISET = 1,NSET
  ERRORINI = 0.0D0
```

```
-----
*      .... INITIAL CONDITIONS ....
```

```
DO JJ = 1,Np
  Y(JJ) = YINI(ISET,1)
  Y(Np+JJ) = YINI(ISET,2)
  Y(2*NP+JJ) = YINI(ISET,3)
  Y(3*NP+JJ) = YINI(ISET,4)
  Y(4*NP+JJ) = YINI(ISET,5)
  Y(5*NP+JJ) = YINI(ISET,6)
  IF(JJ.EQ.NP)THEN
    Y(5*NP+JJ) = 0.00001D0
  ENDIF
END DO
write(6,*)'SIMULATION CALLED'
write(60,*)'SIMULATION CALLED'
```

```
TEMP = 463.16D0
TGINFINITY = 350.0D0
P1 = 0.03D0
ALPHAL = 1.0D-3
ALPHAG = 4.5D-4
RADIUS = 0.02D0
WS = 0.00001D0
```

```
-----A_K PARAMETERS & CONSTANTS-----
```

```
NINDEX = 2.0D0
VMS = 0.864263D0
VPS = 0.81232D0
VSS = 0.969D0
ZETA = 0.709296D0
GAMMA = 1.0D0
VM0 = 1.13247D0
VFM = 0.31033D0
VS0 = 1.1429D0
VFS = 0.1795D0
```

```
WM = 113.16D0
WSOL = 18.0D0
ZETA13 = 0.709296D0
ZETA23 = 0.1265D0
```

```
X = 0.0D0
EK1 = 2.351698D-03
EK2 = DEXP((DS(2)-DH(2)/TEMP)/RR)
EK3 = DEXP((DS(3)-DH(3)/TEMP)/RR)
EK2 = 1033.2474D0
EK3 = 2.456D0
```

```

*      ---
DR = 0.002d0
DO JK = 1,NP
R(JK) = DR*(JK - 1)
END DO
*      ---

```

```

*      .. Executable Statements ..

```

```

c      WRITE (NOUT,*) '--- NYLON 6 SSP SIMULATION ---'
XEND =24.0D0
MPED = 0
IR = 2

```

```

      I = 11
      H = (XEND-X)/DBLE(I+1)
      IFAIL = 0
      ICOUNT = 0
      CALL D02EBF(X,XEND,N,Y,TOL,IR,FCN,MPED,D02EJY,OUT,W,IW,IFAIL)
c      CALL D02CBF(X,XEND,N,Y,TOL,IR,FCN,OUT,W,IFAIL)
      IF (TOL.LT.0.0D0) WRITE (NOUT,*) ' Range too short for TOL'

```

```

      ERROR = ERROR + ERRORINI
777      CONTINUE
55      RETURN
      write(6,*)'F VALUE=', ERROR

```

```

*
99999 FORMAT (1X,A,D8.1)
99998 FORMAT (1X,A,F7.3)
99997 FORMAT (1X,A,6((11E16.5)/))
99996 FORMAT (F4.2/6((11E16.5)/))
END

```

```

*      -----
SUBROUTINE FCN(T,Y,F)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION NINDEX
PARAMETER      (N=66,Np=11)
DIMENSION F(N), Y(N)
COMMON /B2/R(Np),FKS2(Np),FKS3(Np),RKS2(Np),RKS3(Np)
COMMON /B5/DW,TEMP,WS,DR
COMMON /B21/FKS1(NP),RKS1(NP)
common /const/const

```

```

c      .. Executable Statements ..

```

```

DR = 0.002D0

```

```

DO 25 JJ = 1,Np
IJ1      = JJ
IJ2      = 1*Np + JJ
IJ3      = 2*Np + JJ
IJ4      = 3*Np + JJ
IJ5      = 4*Np + JJ
IJ6      = 5*Np + JJ

```

```

CALL RATECONST(JJ,IJ3,IJ4,IJ6,Y)

```

```

      ZZZ = +FKS1(JJ)*Y(IJ1)*Y(IJ6) - RKS1(jj)*Y(IJ2)
      XL3  = Y(IJ5)* ( 2.0D0*Y(IJ5)*Y(IJ3) - Y(IJ4)*Y(IJ4) )
      *    / (Y(IJ4)*Y(IJ3))
      F(IJ1) = -FKS3(JJ)*Y(IJ1)*Y(IJ3)
      *    +RKS3(JJ)*(Y(IJ3) - Y(IJ2)) - ZZZ
      F(IJ2) = -2.0D0*FKS2(JJ)*Y(IJ2)*Y(IJ3)
      *    +2.0D0*RKS2(JJ)*Y(IJ6)*(Y(IJ3) - Y(IJ2))
      *    -FKS3(JJ)*Y(IJ2)*Y(IJ1)
      *    +RKS3(JJ)*const*Y(IJ2) + ZZZ
      F(IJ3) = -FKS2(JJ)*Y(IJ3)*Y(IJ3)
      *    +RKS2(JJ)*Y(IJ6)*(Y(IJ4) - Y(IJ3))
      *    +ZZZ
      F(IJ4) = FKS3(JJ)*Y(IJ1)*Y(IJ3)
      *    -RKS3(JJ)*(Y(IJ3) - Y(IJ2))
      *    +ZZZ
      F(IJ5) = 2.0D0*FKS2(JJ)*Y(IJ4)*Y(IJ4)
      *    +1.0D0/3.0D0*RKS2(JJ)*Y(IJ6)*(Y(IJ4) - XL3)
      *    +FKS3(JJ)*Y(IJ1)*(Y(IJ3) + 2.0D0*Y(IJ4))
      *    +RKS3(JJ)*(Y(IJ3) - 2.0D0*Y(IJ4) + Y(IJ2))
      *    +ZZZ
      IF(JJ.EQ.Np)GO TO 35
      IF(JJ.EQ.1) THEN
        YP = Y(57)
        F(IJ6) = -F(IJ3)
        *    +Dw*((Y(IJ6+1)-2.0D0*Y(IJ6)+YP)/(DR*DR))
      ELSE
        YP = Y(IJ6-1)
        F(IJ6) = -F(IJ3)
        *    +Dw*((Y(IJ6+1)-2.0D0*Y(IJ6)+YP)/(DR*DR))
        *    +Dw*(Y(IJ6+1) - YP)/R(JJ)/DR
      endif
      GO TO 25
35    YP = Y(65)
      F(IJ6) = 0.0D0
25    CONTINUE

      RETURN
      END

```

C=====

```

      SUBROUTINE OUT(X,Y)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION NINDEX
      *    .. Parameters ..
      PARAMETER      (N=66,NP=11)
      PARAMETER      (NOUT=6)
      *    .. Array Arguments ..
      DOUBLE PRECISION Y(N)
      DOUBLE PRECISION RRR(NP)

      COMMON  XEND, H, I
      COMMON /B2/R(Np),FKS2(Np),FKS3(Np),RKS2(Np),RKS3(Np)
      COMMON /B11/RADIUS
      COMMON /B13/TG(NP),VF(NP),DD0(NP),AHUN(NP)

```

```

COMMON /B14/FX1(NP),FX2(NP),FX3(NP),FX4(NP),FX5(NP),FX6(NP)
COMMON /B15/AVEC1,AVEP1,AVELAMO,AVELAM1,AVELAM2,AVEW
COMMON /B16/AVEVF,AVETG
COMMON /B17/FX7(NP),FX8(NP),FX9(NP)
COMMON /B21/FKS1(NP),RKS1(NP)
COMMON /B28/EXMUN(5,13),NSET,ISET
COMMON /B29/ERRORINI,ICOUNT
COMMON /B31/IT,ITMAX

```

```

ICOUNT = ICOUNT + 1
DO JJ = 1,NP
  IJ1 = JJ
  IJ2 = 1*Np + JJ
  IJ3 = 2*Np + JJ
  IJ4 = 3*Np + JJ
  IJ5 = 4*Np + JJ
  IJ6 = 5*Np + JJ
  RRR(JJ) = R(JJ)*R(JJ)
  CALL RATECONST(JJ,IJ3,IJ4,IJ6,Y)
  FX1(JJ) = RRR(JJ)*Y(IJ1)
  FX2(JJ) = RRR(JJ)*Y(IJ2)
  FX3(JJ) = RRR(JJ)*Y(IJ3)
  FX4(JJ) = RRR(JJ)*Y(IJ4)
  FX5(JJ) = RRR(JJ)*Y(IJ5)
  FX6(JJ) = RRR(JJ)*Y(IJ6)
  FX7(JJ) = AMUN(JJ)
  FX8(JJ) = RRR(JJ)*VF(JJ)
  FX9(JJ) = RRR(JJ)*TG(JJ)

```

END DO

CALL AVERAGE(Y,AVEMUN)

WRITE(6,*)'TIME= ',X,' AVEW: ',AVEMUN

IF(IT.GT.(ITMAX-10)) WRITE(21,22)X,AVEMUN

C WRITE(22,23)X,AVEC1,AVEP1,AVELAMO,AVELAM1,AVELAM2,AVEW

22 FORMAT(4X,F10.3,4X,F10.4)

23 FORMAT(4X,F5.2,2X,6(F13.8))

C-----Calculating value of objective function-----

FUN = -ABS((AVEMUN - EXMUN(ISET,ICOUNT)))/AVEMUN

ERRORINI = ERRORINI + FUN

C-----

X = XEND - DBLE(I)*H

I = I - 1

RETURN

*
99999 FORMAT (F10.4/6((6F16.8/5F16.8)/))

END

C=====

SUBROUTINE RATECONST(JJ,IJ3,IJ4,IJ6,Y)

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DOUBLE PRECISION NINDEX


```

PARAMETER      (N=66,Np=11)
DIMENSION Y(N),FKS20(NP),FKS30(NP),RKS20(NP)
COMMON /B2/R(Np),FKS2(Np),FKS3(Np),RKS2(Np),RKS3(Np)
COMMON /B3/EK2,EK3
COMMON /B4/ZETA1,ZETA2,ZETA3
COMMON /B9/THETA,AT,BT,TGINFINITY,P1,ALPHAL,ALPHAG
COMMON /B13/TG(NP),VF(NP),DD0(NP)
COMMON /B20/THETA1,THETA2,THETA3
COMMON /B21/FKS1(NP),RKS1(NP)
COMMON /B22/EK1
COMMON /B23/DD02(np),DD03(np)
COMMON /B25/ZETA13,ZETA23
CALL DIFFUSIVITY1(Y,JJ,1j3,1j4,1j6)
XYZ1 = THETA1/DD0(JJ)
XYZ2 = THETA2/DD02(JJ)
XYZ3 = THETA3/DD03(JJ)

```

```

-----
FKS1(JJ) = ZETA1*Y(IJ3)
FKS20(JJ) = ZETA2*Y(IJ3)
FKS30(JJ) = ZETA3*Y(IJ3)

```

```

-----
RKS1(JJ) = FKS1(JJ)/EK1
RKS20(JJ) = FKS20(JJ)/EK2
RKS3(JJ) = FKS30(JJ)/EK3

```

```

C -----diffusion affected rate constants-----
FKS2(JJ) = 1.0D0/(1.0D0/(FKS20(JJ)) + XYZ1*Y(IJ3))
RKS2(JJ) = 1.0D0/(1.0D0/(RKS20(JJ)) + XYZ2*Y(IJ3))
FKS3(JJ) = 1.0D0/(1.0D0/(FKS30(JJ)) + XYZ3*Y(IJ3))

```

```

RETURN
END

```

```

C=====
SUBROUTINE DIFFUSIVITY1(Y,jj,1j3,1j4,1j6)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION NINDEX
PARAMETER(N=66,NP=11)

```

```

DIMENSION Y(N),DD(NP),DD1(NP),DD2(NP)
COMMON /B5/DW,TEMP,WS,DR
COMMON /B9/THETA,AT,BT,TGINFINITY,P1,ALPHAL,ALPHAG
COMMON /B11/RADIUS
COMMON /B13/TG(NP),VF(NP),DD0(NP),AMUN(NP)
COMMON /B18/VMS,VPS,GAMMA,VMO,ZETA,VFM
COMMON /B19/NINDEX,WM,WSOL
COMMON /B20/THETA1
COMMON /B23/DD02(np),DD03(np)
COMMON /B24/V80,VFS,V8S
COMMON /B25/ZETA13,ZETA23

```

```

AMUN(JJ) = Y(IJ4)/Y(IJ3)
TG(JJ) = 1.0D0/(1.0D0/TGINFINITY + P1/AMUN(JJ))
VF(JJ) = 0.025D0 + (ALPHAL - ALPHAG)*(TEMP - TG(JJ))
DD0(JJ) = DEXP(2.303D0*VF(JJ)/(AT + BT*VF(JJ)))

```

```

VP0 = VPS*(1.0d0+VF(JJ))
TV = (Y(JJ)*WM*VM0 + Y(IJ3)*AMUN(JJ)*WM*VP0 + Y(IJ6)*WSOL*VS0)
PHIM = Y(JJ)*WM*VM0/TV
PHIS = Y(IJ6)*WSOL*VS0/TV
PHIP = Y(IJ3)*AMUN(JJ)*WM*VP0/TV
DM = 1.0d0/VM0
DP = 1.0d0/VP0
DS = 1.0d0/VS0
VFP = VF(JJ)

```

```

XNUM = GAMMA*(DM*PHIM*VMS/ZETA13 + DS*PHIS*VSS/ZETA23 +
*      DP*PHIP*VPS)
XDEN = DM*PHIM*VMS*VFM + DS*PHIS*VSS*VFS + DP*PHIP*VPS*VFP
CHI = XNUM/XDEN

```

```

CHI0 = GAMMA/VFP

```

```

DD(JJ) = DEXP(-CHI + CHI0)
DD02(JJ) = DEXP(ZETA23*(-CHI+CHI0))
DD03(JJ) = DEXP(ZETA13*(-CHI+CHI0))

```

```

DD1(JJ) = AMUN(JJ)*(-NINDEX) * DD(JJ)

```

```

DD0(JJ)=DD1(JJ)
RETURN
END

```

C=====

```

SUBROUTINE AVERAGE(Y,AVEMUN)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DOUBLE PRECISION NINDEX
PARAMETER (N=66,NP=11)
DIMENSION Y(N)
COMMON /B11/RADIUS
COMMON /B13/TG(NP),VF(NP),DD0(NP),AMUN(NP)
COMMON /B14/FX1(NP),FX2(NP),FX3(NP),FX4(NP),FX5(NP),FX6(NP)
COMMON /B17/FX7(NP),FX8(NP),FX9(NP)
COMMON /B15/AVEC1,AVEP1,AVELAM0,AVELAM1,AVELAM2,AVEW
COMMON /B16/AVEVF,AVETG

```

```

DENO = (0.02d0**3.0d0)/3.0d0
CALL INTEGRATION(FX1,0.0,0.02,11,AINT1)
CALL INTEGRATION(FX2,0.0,0.02,11,AINT2)
CALL INTEGRATION(FX3,0.0,0.02,11,AINT3)
CALL INTEGRATION(FX4,0.0,0.02,11,AINT4)
CALL INTEGRATION(FX5,0.0,0.02,11,AINT5)
CALL INTEGRATION(FX6,0.0,0.02,11,AINT6)
CALL INTEGRATION(FX7,0.0,0.02,11,AINT7)
CALL INTEGRATION(FX8,0.0,0.02,11,AINT8)
CALL INTEGRATION(FX9,0.0,0.02,11,AINT9)
AVEC1 = AINT1/DENO
AVEP1 = AINT2/DENO
AVELAM0 = AINT3/DENO
AVELAM1 = AINT4/DENO

```

AVELAM2 = AINT5/DENO
AVEW = AINT6/DENO
AVEVE = AINT8/DENO
AVETG = AINT9/DENO
AVEMUN = AINT4/AINT3

RETURN
END

=====

SUBROUTINE INTEGRATION(FX,XI,XL,NPOINTS,XINT)

THIS SUBROUTINE INTEGRATES FUNCTION 'FX' OVER THE RANGE 'XI' TO 'XL'
'NPOINTS-1' NO. OF INTERVALS. NPOINTS SHOULD BE ODD!
'XINT' IS THE VALUE OF INTEGRAL USING SIMPSONS' RULE.

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
PARAMETER (NP=11)
DIMENSION FX(NP)

N1 = NPOINTS - 1
N2 = NPOINTS - 2
DX = 0.002

SUM1 = 0.0
DO JJ = 3,N2,2
SUM1 = SUM1 + FX(JJ)
END DO

SUM2 = 0.0
DO JJ = 2,N1,2
SUM2 = SUM2 + FX(JJ)
END DO

SUM = FX(1) + FX(NPOINTS) + 2.0D0*SUM1 + 4.0D0*SUM2
XINT = DX*SUM/3.0

RETURN
END

=====